

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 1**

April 2017
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Acknowledgements

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NRS Level 1 Math

Beginning ABE Literacy (Grade Levels 0-1.9)

COUNTING AND CARDINALITY / NUMERACY (CC)

1.CC.1 / 1.CC.2 / 1.CC.3 / 1.CC.4 / 1.CC.5 / 1.CC.6 / 1.CC.7

Essential Understandings:

- Counting determines how many or how much a quantity/number represents.
- When counting, the last number spoken is the total number of objects.
- Counting one more will be the next larger number.
- Each successive number name refers to a quantity that is one larger.
- Knowledge of numbers 0-10 can be applied to predict order and sequence in higher numbers (10-20, 20-30, etc.)
- Quantities of numbers can be compared, ordered, and described as less than, greater than, or equal to one another.
- A written number represents an amount/quantity/order and each number represents a different amount/quantity/order.

Essential Questions:

- Why/when are objects counted? What objects are/can be counted?
- How is number order helpful to us?
- What can numerals represent?
- How would you describe a teen number?
- How can you use 0-10 to predict other counting sequences?

Student will be able to:

(what does mastery look like)

- Count to 100 by ones and tens.
- Count numbers forward from any given number within a learned sequence.
- Write numerals 0-20 and represent a corresponding quantity.
- Use one-to-one correspondence when counting, count numbers in sequence.
- Count objects and know the last number said is the total number of objects counted.
- Name number of objects in a set regardless of position or the order of object count.
- Count and associate each successive number name with a corresponding quantity that is one larger.
- Count up to 20 objects in an orderly arrangement and up to 10 objects in a scattered arrangement.
- Compare and contrast the number of objects within a group of ten by using the phrases “*greater than*,” “*less than*,” and “*equal to*.”
- Compare written numerals between 1 and 10 represented by given written numerals.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Create a graphic representation of level appropriate quantities and assess with rubric
- Teacher observation with rubric of students using math manipulatives to demonstrate level appropriate quantity and symbol connections
- TPR/command cards task performance with rubric
- Teacher observation of small group conversations (e.g., discussing/describe what is a teen number) and projects (applying concept of teen numbers kinesthetically, orally, through audio or visually) using rubric
- Formal teacher observation with rubric

Other Evidence:

- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Math journal

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Coins/Bills for counting by ones and tens (either the unit is by cents OR dollars to avoid decimal numbers)
- Comparing prices on tags (by cents OR dollars to avoid decimal numbers)
- Counting page numbers read in book either from 1 or from first page read
- Counting items/people/places
- Telling which address falls in a block knowing the house number
- Math U See or other manipulative base ten materials for ones, tens and hundreds
- Color-coded cards (ones, tens, hundred) for associating symbol with quantity
- Manipulative bead chains for counting by ones and tens and ticketing with numeric labels by ones and tens
- Textbooks (Contemporary/McGraw Hill) – exercises as assigned
 - *Number Sense: Whole Numbers, Addition & Subtraction*
 - *TABE Fundamentals Mathematics Workbook, Level E*

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- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - Counting dimes, \$10 bills
 - Counting page numbers read
 - Counting items in a group
 - Counting coins to pay for a purchase
 - Telling which address falls in a given block, knowing the first number on the block
 - Counting the number of loose coins in a pile
 - Separating loose coins into like piles and counting the number in each
 - Comparing price tags on two items

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

List of Instructional Materials:

- Coins/Bills (play money or real)
- Counters (discs, beans, buttons, etc.)
- Base Ten manipulatives (Math U See)
- Manipulative bead chains (tens up to hundred)
- Manipulative teens beads (ten bars and unit beads) and boards with numeral tiles
- Manipulative tens beads and boards with numeral tiles

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- Manipulative Hundreds Board
- Manipulative Number Rods (1-10) with numeral cards
- Manipulative spindle box (presents concept of zero along with counting reinforcement)
- Word problem cards with/without authentic materials for solving real life problems and simulations.
- *Number Sense: Whole Numbers, Addition & Subtraction, 2nd ed.* (2003). McGraw Hill Education.
- *TABE Fundamentals Mathematics Workbook, Level E.* (2010). McGraw Hill Education.

List of Technology Resources:

- Kahn Academy – <http://khanacademy.org/>
- Greg Tang Math—www.gregtangmath.com
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

OPERATIONS AND ALGEBRAIC THINKING (OA)

1.OA.1 / 1.OA.2 / 1.OA.3 / 1.OA.4 / 1.OA.5 / 1.OA.6 / 1.OA.7 / 1.OA.8 / 1.OA.9 /
1.OA.10 / 1.OA.11 / 1.OA.12 / 1.OA.13

Essential Understandings:

- Addition and subtraction can be represented by objects, drawings, manipulatives, and other modalities.
- Expressions and equations can be used to decompose numbers in more than one way.
- Quantities can be used to create a variety of individual groupings.
- Numbers less than or equal to 20 can be decomposed by adding, subtracting, or re-grouping.
- The whole is equal to the sum of its parts; conversely, the whole minus a part is equal to the other part.
- Strategies (for example, properties of addition) can be used to decompose complex problems to make an easier problem (counting on, make a ten, near ten, doubles, plus one, plus two, etc.)
- Problem solving structures reinforce part/whole and number combinations within 20
- Word problems have basic problem solving structures: adding to, taking from, putting together, taking apart, comparing and can be represented using different modalities.
- Unknowns can be in various locations (start, change, result) in equations and develop from combinations of numbers.
- Addition and subtraction are related/inverse operations.
- Various strategies can be used to quickly add numbers.
- The equal sign is used to represent quantities that have the same value.

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Essential Questions:

- Why should numbers be decomposed to form different combinations of a specific number?
- What is the connection of a number to an equation or expression?
- How are word problems connected to an equation or expression?
- Why is it important to know multiple strategies in solving addition/subtraction problems?
- How are problem solving strategies and/or properties connected to number relationships?
- What is the relationship between addition and subtraction?
- How can word problems be decoded into equations or expressions to solve them?
- Does a solution make the equation true or false? How is a solution evaluated and does it make sense?

Student will be able to:

(what does mastery look like)

- Represent addition and subtraction through multiple modalities (e.g., manipulatives, actions, drawings).
- Add and subtract within 10 and solve word problems using objects or drawings.
- Represent addition and subtraction as taking apart and putting together pairs of numbers equal to or less than 10 in multiple ways (e.g., using objects such as counters), and draw or write the corresponding equations.
- Starting from any number from 1 to 9, find the unknown number that makes 10 when added to the starting number (e.g., by using objects or drawings), and draw or write the resulting equation.
- Fluently add and subtract within 5.
- Solve addition and subtraction word problems within 20 involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in the start, change and results positions. Use objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- Solve addition word problems using three whole numbers whose sum is less than or equal to 20 using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- Add and subtract by applying the commutative and associative properties of operations, though students need not use formal terms for these properties.
- Subtract within 20 using the relationship between subtraction and addition by solving for the unknown addend (e.g., Solve “ $10 - 8 = ?$ ” by finding “ $8 + ? = 10$ ”).
- Use “counting on” strategy to add and subtract.
- Add and subtract within 20 (fluently within 10), using multiple strategies such as counting on, making 10, decomposing numbers leading to 10 using associative

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property, using relationships between addition and subtraction, and using equivalent/known sums for problem solving.

- Read equations, understanding the function of the equal sign, and determine if an equation is true or false (e.g., Which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$).
- Solve for the unknown whole number in all positions (start, change, result) in an addition or subtraction equation (e.g., Solve for $8 + ? = 11$, $5 = ? - 3$, $6 + 6 = ?$).

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher observation with rubric of student work using manipulatives
- Teacher observation with rubric of group work, discussions on solving for word problems.
- Student creation of and solving for level-appropriate word problems in meaningful contexts (assessed with rubric)

Other Evidence:

- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student journal

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Menus (adapted for coins OR bills to avoid decimal numbers)
- Count change from bill up to \$20.
- Digital clock for counting time.
- Meaningful authentic materials (ex: shopping list items to determine amount needed for recipe)
- Manipulative number rods and short bead stairs (1-9) to show composing/decomposing of numbers to 10.
- Addition board and subtraction grid/board (to establish systematic way for learning addition/subtraction tables)
- Addition chart and subtraction chart (to aid in memorizing addition and subtraction tables)

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- Manipulative bead “snake” game (to reiterate need to return each time to the 1-10 sequence in counting as well as prep for learning addition/subtraction tables)
- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - figuring the number of hours of work or sleep by using fingers to count
 - counting money and making change
 - using manipulatives to establish number relationships
 - working out the shortfall in numbers (e.g., eggs for a recipe, plants to fill a display tray, cups to serve visitors)
 - finding the total price of 3 items ordered from a menu
 - placing same number of cookies on different shaped trays
 - figuring change to receive from a \$10 bill
 - watching a digital clock count down the time
 - paying a \$12 charge with a \$10 bill and two \$1 bills
 - test taking when seeking employment
 - helping children with homework

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction such as for the purpose of additional and varied practice opportunities on targeted skills. As an intervention, see UDL explanation above.

List of Instructional Materials:

- Manipulative number rods
- Manipulative short bead stairs (1-9)
- Manipulative addition/subtraction boards with strips

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- Manipulative addition/subtraction charts with tables, booklets, etc. as well as control charts
- Manipulative addition and subtraction “snake” games
- Simplified menus
- Coins/Bills
- Digital clocks
- Word problem cards with/without authentic materials for solving real life problems and simulations.

List of Technology Resources:

- Khan Academy—<http://khanacademy.org/>
- Greg Tang Math—www.gregtangmath.com
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

NUMBER AND OPERATIONS IN BASE TEN (NBT)

1.NBT.1 / 1.NBT.2 / 1.NBT.3 / 1.NBT.4 / 1.NBT.5 / 1.NBT.6

Essential Understandings:

- Two digit numbers are composed of groups of tens and ones and can be compared with symbols (<, >, =) in terms of their relationship.
- Various models can be used to build individual numbers with tens/ones while counting.
- Counting sequences can be used to understand counting by 10s, identifying 10 more, 10 less.
- Counting can be connected to adding and subtracting.
- Addition can be used to solve and/or evaluate subtraction and vice versa.
- Mental math can be used to check and/or perform calculations in base 10.

Essential Questions:

- How do addition and subtraction relate to counting sequences?
- How does understanding properties of operations help with strategies when performing written and mental calculations?
- How does using objects and drawings help represent problems in multiple ways?
- What is significant about 10?
- What is significant about the teen numbers and how do these numbers relate to 10? (e.g., $10 + 3 = 13$).

Student will be able to:

(what does mastery look like)

- When given a set of objects (ranging from 0-120), read, write, and represent any quantity with a written numeral, and count to 120 starting at any number less than 120.

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- Explain the value of each digit in a two-digit number by identifying a bundle of 10 ones as a “ten,” representing a two-digit numeral using “tens” and “ones,” and representing a two-digit numeral ending in 0 (ranging from 10-90) using “tens” and 0 “ones.”
- Demonstrate understanding of tens and ones digits by comparing two two-digit numbers with comparison words *greater than*, *less than*, and *equal to*.
- Build and decompose numbers into tens and ones (including adding a two-digit and one-digit number and adding a two-digit number and a multiple of 10), relate the chosen strategy (e.g., concrete models, drawings, or place value understanding) for the operation to a written method, and explain the reasoning used.
- Mentally add ten and take away ten to a given two-digit number, and explain the process of doing so.
- Subtract multiples of 10 from decade numbers within 100, relate the chosen strategy (e.g., concrete models, drawings, or place value understanding) for the operation to a written method, and explain the reasoning used.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher observation of student work with manipulatives, using rubric
- Teacher observation of group work and discussions, using rubric.
- Creating/solving level-appropriate word problems in meaningful contexts, assessed with rubric

Other Evidence:

- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student journal

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Checking account deposits (fill out or interpret).
- Count money to make change.
- Count pages read in book.

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- Use manipulative short bead stairs (1-9) and Math U See Base Ten ones/tens beads with Seguin boards and number cards to show place value for teens and tens.
- Use base ten beads of ones/tens/hundred to add/subtract quantities and to show how ones can be composed into tens and applied to addition.
- Use color-coded and sized numeral cards to show how written numbers can be composed/decomposed according to place value.
- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - counting page numbers read at one time, starting from first page read
 - using mental math to check that correct change was received
 - telling which address falls in a block, knowing the house number
 - calculating the production shortfall from a daily target
 - performing mental addition
 - verifying deposits in a checking account
 - counting money and making change
 - changing radio stations

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-on-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
 - Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

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List of Instructional Materials:

- Color-coded and sized number cards according to place value
- Manipulative Base Ten materials for showing ones/tens values
- Play money
- Checking/savings account deposit slips/ledger
- Word problem cards with/without authentic materials for solving real life problems and simulations.

List of Technology Resources:

- Khan Academy—<http://khanacademy.org/>
- Greg Tang Math—www.gregtangmath.com
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

MEASUREMENT AND DATA (MD)

1.MD.1/ 1.MD.2 / 1.MD.3 / 1.MD.4 / 1.MD.5 / 1.MD.6 / 1.MD.7

Essential Understandings:

- Some attributes are measurable; both numbers and words can be used to describe and compare the measurements.
- Objects can be classified, ordered, and compared by attributes and/or measurement.
- Time is measured in hours and half-hours using analog and digital clocks.
- Data can be organized and classified by comparing attributes (height, width and depth).

Essential Questions:

- How are measurable attributes determined and why are these attributes of objects important to comparing quantities?
- How are dividing a circle and telling time related?
- What is the purpose of categorizing data?
- What strategies can be used to organize data?

Student will be able to:

(what does mastery look like)

- Use words to describe several measurable attributes of an object such as length or weight.
- Measure and compare two objects regarding their measurable attributes (e.g., *taller than*, *shorter than*, *heavier than*, *lighter than*).
- Sort items by classification into given categories (e.g., size, color, type), count the items per category, and sort the categories by count.

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- Order three objects according to length and compare two lengths based on the third.
- Select an object as a “length unit” to measure lengths of larger objects (e.g., “The table is 5 dowel rods long.”), lay the selected length units end to end with no gaps between, and count and name the length of the larger object as a whole number unit measure.
- Identify the time by hours and half hours, verbally and in writing, using analog and digital clocks.
- Organize data with up to three categories in various ways, and ask and answer quantity and comparison questions about the categories of data.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher observation of student work with manipulatives, using rubric
- Teacher observation of group work and discussions, using rubric.
- Creating/solving level-appropriate word problems in meaningful contexts, assessed with rubric
- Project-based learning with rubric assessment

Other Evidence:

- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student journal

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

Materials for classroom demonstrations and instructor directed exercises:

- Use manipulative length rods, width rods and stacking tower to present concept of varied measurable attributes
- Cuisenaire rods (length units for measuring other Cuisenaire rods or other objects)-For teacher demonstration/student manipulative.
- Meaningful, authentic materials from daily life (aka “realia”) and images for classification and/or measurement of length, weight, etc.

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- Analog and digital clocks to measure time.
- Flash cards for matching analog time to digital time according to hour/half hour units of time.
- Analog clock stamp for writing in hands for hours/half hours.
- A “judy” clock with gears to move hands in response to oral prompt or use of clock as prompt for written work.
- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - describing a rectangular photo or frame
 - describing seasons, daylight savings time or tides
 - sorting laundry or bottles for the recycling facility
 - understanding a child’s growth chart
 - reading a bus schedule

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-on-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

List of Instructional Materials:

- Manipulative red length rods
- Manipulative brown stairs
- Manipulative pink tower
- Cuisenaire rods
- Meaningful authentic materials from daily life (aka “realia”) and images for classifying and/or measuring
- Analog and digital clocks.

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- “Judy” clock with gears (or Jude e-clock at www.mrmyers.org)
- Flash cards for matching analog time to digital time according to hour/half hour units of time.
- Analog clock stamp and pad
- A bus schedule
- Child’s growth chart (length/weight)
- Word problem cards with/without authentic materials for solving real life problems and simulations

List of Technology Resources:

- Khan Academy—<http://khanacademy.org/>
- Greg Tang Math—www.gregtangmath.com
- CK-12 <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

GEOMETRY (G)

1.G.1 / 1.G.2 / 1.G.3 / 1.G.4 / 1.G.5 / 1.G.6 / 1.G.7 / 1.G.8 / 1.G.9

Essential Understandings:

- Objects have position relative to other objects using terms such as “above,” “below,” “beside,” “in front of,” “behind,” and “next to.”
- Two-dimensional shapes are flat and can be built from components.
- Three-dimensional shapes have unique attributes and specific names regardless of their orientations or overall size.
- Attributes are used to compare and analyze two- and three-dimensional shapes.
- Circles and rectangles can be used to create more complex shapes; circles and rectangles can be partitioned into equal shares.
- Shapes can be used to build pictures, designs and other shapes.
- Understanding of shapes and components to recognize and represent shapes in the world.

Essential Questions:

- Why are positional words important in math?
- How can shapes be partitioned into halves and quarters?
- Why is mathematical language critical when describing two-dimensional and three-dimensional shapes?
- How can two-dimensional shapes be decomposed or combined to form two- or three-dimensional shapes and vice versa?

Student will be able to:

(what does mastery look like)

- Name and describe shapes in the environment, and state the positions of shapes in the environment using prepositions of location.

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- Name shapes regardless of size or orientation.
- Distinguish between planes (two-dimensional shapes) and solids (three-dimensional shapes).
- Analyze and compare planes and solids using informal terms (e.g., counting number of sides and vertices, identifying sides of equal length vs. differing lengths) using informal language.
- Model shapes using shape components or through drawing.
- Create larger shapes from simple shape components (e.g., triangles combined to make rectangles or hexagons).
- Distinguish which attributes of a shape are defining compared to attributes that are non-defining by using models or pictures, and build and draw shapes based on defining attributes.
- Build two- and three-dimensional composite shapes from other shapes.
- Divide circles and rectangles into two and four equal parts using the words halves, fourths, and quarters to describe them, put the pieces back together to make a whole and describe this whole as 2 halves or 4 fourths, and identify that more equal shares lead to smaller shares.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher observation with rubric of student work using geometry manipulatives to create/solve problems
- Teacher observation with rubric of group work and project-based learning.
- Teacher formal assessment of student creation of and/or solving for level-appropriate word problems in meaningful contexts

Other Evidence:

- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student journal

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

Materials for classroom demonstrations and instructor directed exercises:

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- Manipulative geometry cabinet and solids to name, describe, and distinguish two- and three dimensional shapes.
- Manipulative geometry three-part classification cards (picture card, name card, control card with picture and name).
- “I spy” game for shapes in environment using prepositions of location.
- Manipulative geometry sticks to model plane shapes.
- Manipulative constructive triangles to compose larger shapes from triangles.
- Clay, origami, drinking straws, toothpicks or wooden blocks for sculpting shapes and form those shapes into composite shapes. These materials also useful for building a three-dimensional shape from two-dimensional shapes.
- Project learning by building furniture.
- Using shapes to build dioramas or scale models
- Manipulative fraction circles and equivalence material to grasp concept of two or four equal shares are halves and fourths.
- Word problem cards with/without authentic materials for solving real life problems and simulations using the following:
 - recognizing the shape and meaning of a triangular yield sign and other shapes in buildings and everyday structures
 - identifying things by shape
 - building a 3D model from flat (plane) materials
 - using a “rules of the road” book to describe shapes of road signs
 - using shapes to replicate campus map and roadways
 - checking for quality control
 - building furniture
 - cutting pizza, cake, and brownies

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as

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additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.

- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

List of Instructional Materials:

- Manipulative Geometry Shapes Cabinet
- Manipulative Solid Geometry Shapes
- Manipulative Geometry Classification cards (picture/name/control of error or picture/definition/control of error)
- Manipulative Geometry Sticks
- Manipulative Constructive Triangle material
- Manipulative Fraction Circle material and Geometric Fraction Equivalence Material
- Pizza/brownies/cake in round and square pans
- Clay, origami, drinking straws, toothpicks, wooden blocks
- Word problem cards with/without authentic materials for solving real life problems and simulations using shapes two- and three-dimensional found in the classroom/real world environment

List of Technology Resources:

- Khan Academy—<http://khanacademy.org/>
- Greg Tang Math—www.gregtangmath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 2**

April 2017
REVISED June 2021

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NRS Level 2 Math
Beginning Basic ABE (Grade Levels 2.0 – 3.9)

OPERATIONS AND ALGEBRAIC THINKING (OA)

2.OA.1 / 2.OA.2 / 2.OA.3 / 2.OA.4 / 2.OA.5 / 2.OA.6 / 2.OA.7 / 2.OA.8 / 2.OA.9 /
2.OA.10 / 2.OA.11 / 2.OA.12 / 2.OA.13

Essential Understandings:

- There are different problem solving structures that can be used to solve problems in multiple ways.
- Unknown quantities can be represented in different places in an equation/number model.
- Addition and subtraction can be represented on various models such as number lines, picture graphs, algebra tiles, and bar graphs.
- Word problems can be structured to require multi-step solutions.
- Fluency with all sums, differences, products, and quotients of two numbers (0-12).
- Even numbered objects can be modeled using pairs or rectangular arrays.
- The difference between even and odd numbers.
- Visual images and numerical patterns of multiplication and division can be used in problem-solving situations.
- The Properties of Operations will help in performing computations as well as in problem-solving situations (Distributive, Associative, Commutative, Identity, and Zero.)

Essential Questions:

- How does an equation represent an unknown quantity?
- How do visual representations depict and help solve addition, subtraction, multiplication, and division problems?
- How does fluency with basic sums, differences, products, and quotients help in problem solving situations?
- What are efficient methods for finding sums and differences using even and odd properties of numbers?
- How do multiples and factors relate to multiplication and division?
- How can inverse operations be used to solve problems?
- How can the reasonableness of a solution be evaluated?
- How can arithmetic patterns be used to help find solutions to problems?
- What are some of the rules or properties of whole numbers?

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Student will be able to:

(What does mastery look like)

- Solve one- and two-step word problems by adding and subtracting within 100 involving situations of adding to, taking from, putting together, taking apart, and comparing, and solve these problems with unknowns in the start, change, and results positions.
- Use mental strategies to add and subtract fluently within 20, having memorized all sums of two one-digit numbers.
- Decide if a group of objects up to 20 is odd or even, and show an equation in which equal addends result in even number sums.
- Find the total number of objects arranged in rectangular arrays (up to 5 by 5) by using repeated addition, and write an equation to represent the repeated addition.
- Interpret products of whole numbers as the total number of objects in “so many” groups of “so many” objects each.
- Demonstrate understanding of whole-number quotients of whole numbers by describing the number of objects in each share or as a number of equal shares.
- Multiply and divide for word problems within 100 using equal groups, arrays, and measurement quantities.
- Solve for the unknown whole numbers in all positions (start, change, result) for multiplication and division problems.
- Apply commutative, associative and distributive properties of operations to multiply and divide, though students need not use formal terms for these properties.
- Relate multiplication to division by solving division problems as unknown factor multiplication problems.
- Apply a variety of strategies to multiply and divide fluently within 100, having memorized all sums of two one-digit numbers.
- Solve two-step word problems (limited to whole numbers) using the four operations and represent the word problems as equations with letters representing unknowns, determining reasonableness of answers using mental computation and estimations such as rounding.
- Identify and explain arithmetic patterns using the properties within all four operations.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Using manipulatives such as number beads and chains to demonstrate solving for operations formally assessed by teacher using rubric
- Project-based group work with teacher rubric assessment of process and product.

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- Creating/solving level-appropriate word problems in meaningful, real-world contexts using word problem prompts and/or authentic real-life materials/situations for formal assessment with rubric.
- Using teacher-made worksheets

Other Evidence:

- Interactive math journals
- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student demonstrations or presentations
- Project-based learning
- Simulations

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Snake Game using beads for addition and subtraction practice and memorization
- Addition, Subtraction, Multiplication and Division manipulative boards and charts to facilitate fact memorization
- Manipulative Bead Bars to show multiples of 1 to 10, to show relationship between multiplication and division, and to use with operation symbols tickets and cut-out parentheses in order to show commutative, associative and distributive properties of multiplication
- Manipulative Racks/Tubes Division Board (for Unit Divisor only) and Math U See Base Ten Materials to show distributive division/sharing out principle
- Manipulative Tiles Game to show group division
- Hundreds Board and Pythagoras/Multiples Boards to facilitate memorization of addition and multiplication facts and to see patterns in addition and multiplication tables.
- paper, pencil, colored pencils to draw out problems and equations
- Textbooks & workbooks (Contemporary/McGraw Hill Materials: *Achieving TABE Success in Math, Level E, Number Power 1, Number Sense 1 & 2*)

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- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - carrying out a stock inventory
 - checking grocery receipt against purchases
 - estimating the bill at a restaurant
 - telling which side of a street a house will be on from its number
 - finding out how many chairs are needed for a meeting
 - determining how many pieces of pie you will have with multiple pies
 - splitting a restaurant bill (check) into equal parts for 2, 3, 4, 5 or more people
 - determining the total amount of money when each person pays an equal fee
 - calculating total number (e.g., three days a week for four weeks)
 - working out how many cars are needed to transport a group of people
 - dividing work time for employees
 - estimating amount of purchase to nearest 10 dollars
 - estimating distances between cities
 - giving ballpark figures for numbers in a crowd
 - laying tile on a floor.

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.

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- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills

List of Instructional Materials:

- Manipulatives: Bead Box 1-10 along with number tickets, operations tickets, parenthesis tickets
- Manipulative Addition/Subtraction/Multiplication/Division Boards (variations & controls) and charts
- Manipulative Base Ten (Math U See) Material
- Manipulative Racks/Tubes Board
- Hundreds and Pythagoras Boards
- Graph paper and pencil/eraser/colored pencils
- Word problem cards

List of Technology Resources:

- Khan Academy -- <http://khanacademy.org/>
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

NUMBERS AND OPERATIONS IN BASE TEN (NBT)

2.NBT.1 / 2.NBT.2 / 2.NBT.3 / 2.NBT.4 / 2.NBT.5 / 2.NBT.6 / 2.NBT.7 / 2.NBT.8 /
2.NBT.9 / 2.NBT.10 / 2.NBT.11

Essential Understandings:

- Numbers are composed of other numbers.
- Numbers can represent quantity, position, location and relationships.
- Place value is based on groups of ten.
- Flexible methods of computation involve grouping numbers in strategic ways.
- There are different problem solving structures that can be used to solve problems in multiple ways.
- Strategies based on place value and properties of operations can be used to represent the product of one digit whole numbers by multiples of 10 (in the range of 10-90).

Essential Questions:

- How can numbers be expressed, ordered and compared?

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- How does the position of a digit in a number affect its value?
- In what ways can numbers be composed and decomposed using addition, subtraction and multiplication?
- What are efficient methods for finding sums and differences?

Student will be able to:

(what does mastery look like)

- Represent three digit numbers as amounts of hundreds, tens, and in a variety of ways, including representing 100 as a bundle of 10 tens, and representing 100, 200, 300, 400, 500, 600, 700, 800 and 900 as the appropriate number of hundreds.
- Demonstrate comprehension of place value by skip-counting by 5s, 10s, and 100s within 1000.
- In standard, expanded, and number name form, read and write numbers to 1000.
- Compare two three-digit numbers based on place value of each digit, using comparison symbols $>$, $<$, and $=$.
- Using strategies of place value and properties of operations, add up to four two-digit numbers.
- Add and subtract within 1000 using models, drawings, operation properties and/or the relationship between addition and subtraction using base 10 strategies, relating the chosen strategy to a written method.
- Mentally add or subtract 10 or 100 to or from a given number 100-900.
- Demonstrate how addition and subtraction strategies work by applying knowledge of place value and the properties of operations using concrete objects, pictures and words.
- Round whole numbers to the nearest 10 or 100 to demonstrate knowledge of place value.
- Add and subtract fluently within 1000 using strategies related to place value, operations properties, and the relationship between addition and subtraction.
- Apply strategies of place value and operations property by multiplying one-digit whole numbers by multiples of 10 in the 10-90 range.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Using math manipulatives such as Math U See Base Ten materials to demonstrate concept recognition and solving for operations formally assessed by teacher using rubric
- Project-based group work with teacher rubric assessment of process and product

NRS Level 2 Math Beginning Basic ABE (Grade Levels 2.0 – 3.9)

- Creating/solving level-appropriate word problems in meaningful, real-world contexts using word problem prompts and/or authentic real life materials/situations for formal assessment with rubric

Other Evidence:

- Interactive math journals
- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student demonstrations or presentations
- Project-based learning
- Simulations

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Use manipulatives such as Math U See Base Ten materials along with color-coded decimal hierarchy number cards to work on quantities 1-1000, numeric representation of quantities, and association and synthesis of quantity to symbol, to add/subtract, including materials to demonstrate composing/decomposing of numbers by place value in doing operations.
- Manipulative long bead chain up to 1000, used with number tickets to practice skip counting.
- Bead frame/abacus designed for decimal categories along with bead frame paper to work more symbolically on place value in adding, subtracting and multiplying up to thousands place value, including decomposing and exchanging numbers by decimal categories in performing addition, subtraction and multiplication operations.
- Activities and exercises in textbooks: *Achieving TABE Success in Math Level E*, *Number Power 1*, and *Number Sense Whole Numbers: Addition and Subtraction*
- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - exchanging money into small bills or coins
 - reading graphs accurately
 - writing a rent or tuition check

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- balancing a checkbook
- finding amount of money remaining after shopping at different places
- determining the new temperature for baking with recipe changes

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

List of Instructional Materials:

- Base Ten Math U See materials (ones/tens/hundreds/thousands)
- Decimal hierarchical number cards (ones/tens/hundreds/thousands) color-coded
- Long Bead Chain (1000) with number tickets for skip counting by 5s, 10s, 100s
- Bead Frame/abacus with ten beads per wire strand representing ones/tens/hundreds/thousands decimal categories along with corresponding Bead Frame paper
- Word problem cards
- authentic materials from real world (e.g., checkbooks, receipts)
- graph paper/pencil (to assist with alignment of categories in operations and in expanded notation of numbers)

List of Technology Resources:

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- Khan Academy -- <http://khanacademy.org/>
- Greg Tang Math -- www.gregtangmath.com
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

NUMBERS AND OPERATIONS IN FRACTIONS (NF) 2.NF.1 / 2.NF.2 / 2.NF.3

Essential Understandings:

- The size of the fractional part is relative to the size of the whole.
- Fractions are quantities where a whole is divided into equal-sized parts and can be represented by models (such as, rulers, manipulatives, words, and/or number lines, etc.)
- Fractions can be used as a tool to understand and model quantities and relationships.
- Fractions are composed of unit fractions.
- Fractions that represent equal-sized quantities are equivalent.
- Two fractions with the same numerator represent the same number of parts.
- Two fractions with the same denominator represent the same number of parts of the whole.
- Whole numbers can be represented as a fraction such as $3 = 3/1$. or any fraction whose numerator and denominator are the same is equal to 1, such as, $4/4 = 1$.

Essential Questions:

- What do fractions represent?
- What makes fractions equivalent?
- What is the relationship between two fractions with the same numerator or two fractions with the same denominator?

Student will be able to:

(what does mastery look like)

- Demonstrate understanding that a fraction represents a part of a whole.
- Demonstrate understanding of fractions by representing them on a number line.
- Compare fractions with respect to equivalency by reasoning about their size, considering fractions that are on the same point on a number line and fractions with the same numerator or denominator.
- Demonstrate understanding of fractions by writing equivalent fractions and explaining why they are equal with a visual model, and by expressing whole numbers as fractions.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Create a graphic representation of a fraction
- Use correct mathematical symbols to compare fractions
- group work/monitoring conversations during group work with rubric
- Using manipulatives – watching students as they work and formally assessing with rubric

Other Evidence:

- Interactive math journals
- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student demonstrations or presentations
- Project-based learning
- Simulations

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Fraction packets
- Hershey Bars
- Groups with recipes, mix up the measuring cups & spoons – then mix up the equipment (students have to figure out equivalencies)
- manipulative fraction circles from one whole to tenths, with tickets for labeling
- Textbooks & workbooks (Contemporary/McGraw Hill Materials: *Achieving TABE Success in Math Level E, Number Power 2, Number Sense Fractions: The Meaning of Fractions*)

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- Word problem cards with/without authentic materials for solving real life problems and simulations.

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills

List of Instructional Materials:

- Manipulative fraction circles with tickets for fraction sections
- Worksheets to use for finding fraction equivalencies
- Other manipulatives such as Hershey bars
- Measuring cups/spoons for use in recipes calling for fraction of 1 cup of ingredients
- Money (quarter is $\frac{1}{4}$ a dollar, etc.)

List of Technology Resources:

- Khan Academy -- <http://khanacademy.org/>
- Greg Tang Math -- www.gregtangmath.com
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

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MEASUREMENT AND DATA (MD)

2.MD.1 / 2.MD.2 / 2.MD.3 / 2.MD.4 / 2.MD.5 / 2.MD.6 / 2.MD.7 / 2.MD.8 / 2.MD.9 /
2.MD.10 / 2.MD.11 / 2.MD.12 / 2.MD.13 / 2.MD.14 / 2.MD.15 / 2.MD.16

Essential Understandings:

- There is a relationship between estimation and measurement.
- Measurement is a way to describe and compare objects or ideas.
- A specific process or tool (i.e., a metric or standard ruler) can be used to measure attributes of unit length.
- Metric measurement units are related to place value concepts/multiples of 10.
- A number line is used to represent measurement attributes such as, distance and quantity.
- Currency has different values and is counted according to its values.
- Standard units provide common language for communicating time.
- Equivalent periods of units are used to measure time.
- Information can be represented in scaled bar and picture graph form. These graphs can be used to help solve one and two- step math problems.
- Elapsed time is the interval of time, given a specific unit, from a starting time to an ending time.
- Perimeter and addition are related.
- A linear unit is used to measure perimeter.
- Everyday objects have a variety of attributes, each of which can be measured in many ways.
- Area can be a function of addition as well as multiplication.
- Perimeter and area are related.
- Modeling (tiling) multiplication and decomposing problems based upon their problem-solving structure can help in finding solutions.
- The mass (two-dimensional figures) and volume (three-dimensional figures) of a substance or solid can be measured and expressed in terms of standard units (square or cubic units).

Essential Questions:

- When is it appropriate to estimate and when is it appropriate to provide an exact answer?
- What properties or attributes can be measured?
- How are attributes measured (unit, tool, and process)?

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- How can accurate measurements solve problems and make sense of the world?
- How does monetary value affect how money is counted?
- How do units within a system relate to each another?
- How are various representations of time related?
- How can understanding the relationship between addition and subtraction aid in problem solving?
- How can data represented in scaled bar and picture graphs be useful in the real world?
- What conclusions can be made about elapsed time and its usefulness?
- How can understanding the relationship between addition and area aid in problem solving?
- How can modeling multiplication and decomposing problems help in finding their solutions?
- What is the relationship between area and addition/multiplication?
- How does metric measurement connect to multiples of 10?
- How does volume or mass of a three-dimensional figure differ from the area of a two-dimensional figure? (Describe in terms of units and/or attributes of each figure.)

Student will be able to:

(what does mastery look like)

- Choose and use an appropriate tool to measure the length of an object.
- Measure the length of an object twice using two different standard units of measure and describe the relationship between the size of the units and the object.
- Correctly estimate lengths by imperial (inches/feet) and metric systems (centimeters/meters).
- Compare objects side by side and measure the difference, expressing the difference in length in terms of a standard unit.
- Solve addition and subtraction word problems for lengths within 100 given in the same unit measure, using drawings and equations, and solving for an unknown number.
- Demonstrate understanding of length by representing whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to positive numbers and by representing whole-number sums and differences within 100 on a number line.
- Identify and write time using analog and digital clocks to the nearest five minutes, correctly identifying a.m. and p.m.
- Demonstrate knowledge of money by solving word problems involving dollars and cents, using the \$ and ¢ symbols correctly.

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- Draw and label a scaled picture graph and a scaled bar graph to represent a data set, and use that information to solve one- and two-step “*how many more*” and “*how many less*” problems.
- Measure objects using rulers marked with halves and fourths of an inch to generate a data set, and then represent that data using a line plot marked by wholes, halves and fourths of inches.
- Demonstrate understanding of time by telling and writing time and time intervals to the nearest minute, and solve addition and subtraction word problems regarding time intervals in minutes.
- Demonstrate understanding of volume and mass by measuring and estimating liquid volumes and masses of objects in standard units of grams (g), kilograms (kg), and liters (l), and solving one-step word problems involving masses or volumes given in the same units (e.g., drawings such as a beaker with a measurement scale).
- Identify and describe square units and explain how square units are used to determine the area of a plane figure.
- Count standard square units (square cm, m, in, ft) to find the area.
- Demonstrate understanding of the relationship between area and the operations of multiplication and addition by tiling in square units the area of a rectangle and showing that same units squared can be found by multiplying the side lengths (base x height).
- Demonstrate understanding of the relationship between area and the operations of multiplication and addition by multiplying the side lengths of rectangles to find the area in real world problems.
- Demonstrate understanding of the relationship between area and the operations of multiplication and addition by tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$, using area models to represent the distributive property.
- Demonstrate understanding of the relationship between area and the operations of multiplication and addition by finding areas of non-overlapping rectangles by decomposing those figures into rectangles to solve for area.
- Apply understanding of perimeter by solving real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding for an unknown side length, and comparing rectangles with the same perimeters/different areas and different perimeters/same areas.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Create graphic/figure representation of measurement (plotting/operations/conversions)

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- Facilitating group work/monitoring conversations during group work
- Using manipulatives – watching students as they work
- Project-based learning--completion of project

Other Evidence:

- Interactive math journals
- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student demonstrations or presentations
- Project-based learning
- Simulations

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Measure aspects of the environment (wall, hallway, table, etc.) with a stick cut to a meter length but unmarked to draw attention to the concept of unit measures and approximation and beginning estimation, and then switch to a standard meter stick with centimeters marked off to as intro to standard measures, including precise measurement to nearest cm, inch, 1/4 inch, etc.
- Use and compare different measures (metric and imperial systems as well as different units of measurement within those systems, such as inches/feet) for measuring the same object. Can create a game, such as, "I'm thinking of an object that is around 2 m. long. How many feet?"
- Use and compare the same unit of measurement to measure for different objects. Can play I Spy game such as, "I'm thinking of an object in this room that is approximately 3 feet long. What is it?"
- Create graphic representations of for plotting/adding/subtracting the measurement for objects.
- Project-based learning (build a table, build a scale model) using measurement.
- Geometric solids/models for measuring volume
- Teacher-made Area Material manipulative made from 1-inch square graph paper cut into rectilinear figures to show the decomposing of rectilinear figures into

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rectangles with non-overlapping unit squares to find area, then applying to real world situations (ex: find area of non-rectangular rooms to find area for laying carpet)

- Manipulative polygon materials to find perimeters and area and relationships/differences
- Personal timelines (day or week) to read write time and measure time intervals in daily/weekly routine
- Use play money for manipulatives in doing operations with dollars and cents
- Textbooks & workbooks (Contemporary/McGraw Hill Materials: *Achieving TABE Success in Math Level E, Number Power 1, 2, 3, 4,*)
- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:
 - measuring a room for carpet
 - sorting by size
 - hanging a picture or an award on the wall
 - deciding whether a sheet of wall paper is long enough for a wall
 - model drawings
 - computing hours worked or to pay a babysitter
 - checking bus schedules in a.m. and p.m.
 - following a recipe
 - buying carpeting, tiles or wallpaper
 - figuring the amount of molding needed around a window

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.

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- Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

List of Instructional Materials:

- Rulers, meter sticks, yard sticks
- Measuring cups and spoons for cooking projects and measuring volume
- Manipulative Area Material made from 1-inch square graph paper
- Polygon Manipulative Material
- Geometric Solids
- Graph paper, pencil
- Authentic/real life materials and objects for project-based learning (make a planter box!)
- Play money
- Adding paper for making personal timelines
- "Judy" clocks with moving gears
- Word problem cards with/without objects/images to solve for real life problems

List of Technology Resources:

- Khan Academy -- <http://khanacademy.org/>
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

GEOMETRY (G)

2.G.1 / 2.G.2 / 2.G.3 / 2.G.4 / 2.G.5

Essential Understandings:

- Any geometric figure can be composed or decomposed from/into other figures whose areas are the sum of its parts.
- Some objects can be described and compared using their geometric attributes (which may be fractional components).

NRS Level 2 Math Beginning Basic ABE (Grade Levels 2.0 – 3.9)

Essential Questions:

- How can the attributes of any geometric figure be composed or decomposed to represent or model the sum of its parts?
- What is the significance of composing or decomposing a geometric figure into the sum of its parts?
- How can plane (two-dimensional) and solid (three-dimensional) shapes be described?

Student will be able to:

(what does mastery look like)

- Name and draw shapes with specified attributes, such as triangles, quadrilaterals, pentagons, hexagons, and cubes, comparing sizes directly or visually as opposed to measuring.
- Section a rectangle into same size squares creating rows and columns and count the number of tiles to determine the total number of squares in the rectangle.
- Section circles and rectangles into two, three, or four equal parts, describing the parts as halves, thirds, and fourths and identifying the combinations of the whole as two halves, three thirds, etc., recognizing that equal sized sections of identical wholes need not have the same shape.
- Compare and contrast shapes in different categories that share attributes and belong to a larger category (e.g., quadrilaterals), and identify and draw examples of more complex quadrilaterals that do not belong to one of a subcategory such as squares, rectangles, or rhombuses.
- Divide shapes into sections with equal areas and write the area of each part as a unit fraction of the whole.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Using manipulatives to demonstrate solving for operations formally assessed by teacher using rubric
- Project-based group work with teacher rubric assessment of process and product.
- Creating/solving level-appropriate word problems in meaningful, real-world contexts using word problem prompts and/or authentic real life materials/situations for formal assessment with rubric.
- Create a graphic/figure representation geometric concepts

NRS Level 2 Math Beginning Basic ABE (Grade Levels 2.0 – 3.9)

Other Evidence:

- Interactive math journals
- Informal teacher observation
- Exit tickets
- Monitoring group work and student discussions
- Student self-assessment
- Student portfolio
- Student demonstrations or presentations
- Project-based learning
- Simulations

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Use manipulative Geometry Sticks and Constructive Triangles materials to construct shapes with specific attributes and how shared attributes of different shapes can define a larger category.
- Use Geometry plane figures manipulatives and Geometry Solids (three-dimensional figures such as cubes) to identify shapes with specific attributes.
- Use teacher-made Area Material made from 1-inch graph paper to partition rectangular shapes into equal shares by row/columns (leads to understanding relationship between unit squares and area--see measurement activities above)
- Manipulative Fraction Circles and Equivalency Materials to understand concepts of the whole as two halves, three thirds, etc. and to grasp concept that equal shares of equivalent whole may not have the same shapes.
- Geometry Nomenclature materials to demonstrate and research the varieties of shapes in broader categories such as "quadrilaterals". These include matching image, Tier 3 terminology, and definition, and includes a control of error sheet to check work.
- Graph paper/pencil/colored pencils/protractor/ruler/geometry compass/origami/clay for drawing and forming own figures for exploration of level appropriate geometry concepts.
- Textbooks & workbooks (Contemporary/McGraw Hill Materials: *Achieving TABE Success in Math Level E, Number Power 4: Geometry*)
- Word problem cards with/without authentic materials for solving real life problems and simulations such as the following:

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- creating a pattern for laying tiles
- making flash cards
- cutting a pie or cake into equal parts
- drawing and sculpting objects with clay
- splitting a pizza or pie into equal slices

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutors
- Pair up for peer-teaching
- Change the frame of reference (objects, images, art and architecture in classroom or on field trip viewed geometrically)
- Universal Design for Learning: this is not an activity itself, but all activities can be designed for broader or “universal” access. The 3 components of UDL include:
 - Multiple representations of information such as text and numbers read aloud by a computer, captions on video, text accompanying audio, verbal descriptions of pictures, drawings and models, and using video, animation or other educational technology to convey concepts.
 - Multiple representations of expression such as students being able to record oral speech (to text or not), drawing, dramatic presentations, songs/raps.
 - Varied options of engagement enabling students as well as teachers to select for or reframe elements of content/context/procedures, challenge level, and supports including modifications and accommodations such as additional time, enlarged print, text-to-speech apps, bilingual dictionary, etc.
- Interactive technologies assigned by instructor to support instruction for the purpose of additional and varied practice opportunities on targeted skills.

List of Instructional Materials:

- Geometry manipulative materials including Geometry Sticks, Constructive Triangles, Geometry shapes for plane figures (2D) and solid figures (3D) Fraction Circles, Equivalency Materials, Geometry Nomenclature Materials and Geometry Charts.
- Graph paper/pencil/colored pencils/protractor/ruler/geometry compass/origami/clay
- Word problem cards with/without authentic materials for solving real life problems and simulations.

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List of Technology Resources:

- Khan Academy -- <http://khanacademy.org/>
- Greg Tang Math -- www.gregtangmath.com
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 3**

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OPERATIONS AND ALGEBRAIC THINKING (OA)

3. OA.1 / 3.OA.2 / 3.OA.3 / 3.OA.4 / 3.OA.5 / 3.OA.6 / 3.OA.7 / 3.OA.8/3.OA.9

Essential Understandings:

- Flexible methods of computation involve grouping numbers in strategic ways.
- The distributive property is connected to the area model and/or partial products method of multiplication.
- Some division situations will produce a remainder, but the remainder should always be less than the divisor. If the remainder is greater than the divisor, that means at least one more can be given to each group (fair sharing) or at least one more group of the given size (the dividend) may be created. When using division to solve word problems, how the remainder is interpreted depends on the problem situation.
- Number or shape patterns are generated by following a given rule.
- The four operations (addition, subtraction, multiplication, and division) are interconnected.
- The greatest common factor (GCF) and the least common multiple (LCM) among whole numbers can be determined.
- The sum of two whole numbers between 1 and 100 can be expressed as a multiple of a sum of two whole numbers (e.g., the distributive property).
- Parentheses, brackets, and braces are used to guide the order of operations when simplifying expressions.
- A standard algorithm is used to fluently multiply multi-digit whole numbers.
- A variety of different strategies can be used to multiply and divide multi-digit numbers including: visual models (rectangular array, equations, and/or area model).
- Strategies for multiplication and division are based on place value, the properties of operations, and/or the relationship between multiplication and division (approaching problems with unknown product or quotient, group size unknown and number of groups unknown).

Essential Questions:

- How do I determine the factors of a number?
- What is the difference between a prime and composite number?
- How are multiplication and division related to each other?
- What are efficient methods for finding products and quotients, and how can place value properties aid computation?
- How are dividends, divisors, quotients, and remainders related?
- How are the four operations of addition, subtraction, multiplication and division used in multi-step word problems? (How can these operations be used to assess the reasonableness of a solution?)
- What is the difference between the GCF and LCM?
- How can the distributive property be used to express the sum of two whole numbers [e.g., $25 + 10$ as $5(5 + 2)$]?
- How can a remainder be interpreted with respect to the answer in a division word problem? (Is the solution reasonable?)
- How do parentheses, brackets, and braces affect the way expressions are simplified or evaluated?
- When are different strategies appropriate to use when multiplying and/or dividing multi-digit numbers?

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- What strategies can be used to find rules for patterns and what predictions can the pattern support?

Student will be able to:

(What does mastery look like?)

- Interpret and represent verbal statements of multiplicative comparisons as equations.
- Solve multiplicative comparison word problems, choosing the operations of division and multiplication (as distinguished from additive comparison), using pictures or variables for the unknown.
- Answer multi-step problems with whole numbers using the four operations to solve for a variable (unknown quantity) and assessing the reasonableness of the solution obtained.
- Determine the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
- Use the distributive property to express the sum of two whole numbers 1-100 with a common factor as a multiple of two whole numbers with no common factor.
- Identify and find all factor pairs for a whole number, recognize multiples of whole numbers, and identify a given whole number as prime or composite in the range 1-100.
- Outline a number or shape pattern, analyze and explain features of the pattern, and identify features of the pattern that are not apparent in the rule itself.
- Write numerical expressions that contain parentheses, brackets, or braces and solve expressions containing them.
- Construct basic numerical expressions with the four operations and interpret them without evaluating them.
- Write two different basic numerical patterns using two different rules.
- Create ordered pairs using corresponding terms from the number patterns, identify the relationship between the corresponding terms, and graph the ordered pairs on a coordinate plane.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Using teacher made worksheets-graded.
- Use teacher made quizzes/exams.
- Discuss a variety of ways to solve multistep problems.

Other Evidence:

- Question and answer sessions.
- Facilitate group work with teacher monitoring.

Building the Learning Plan

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Sample Classroom Activities and/or Lesson Plans:

- Demonstrate and practice partitioning strategies for multiplication.
- Discuss and discover invented strategies for solving multiplication and finding reasonable solutions.
- The Frayer Model (visual organizer) Demonstrate and practice.
- Artistic illustrations of different shape patterns
- Creating cue cards for various types of math problems
- Practice factoring whole numbers
- Practice using parenthesis, brackets and braces in mathematical expressions and solving expressions containing them with teacher-generated examples.

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one intervention
- Peer tutoring
- Think out loud (demonstrate how to think about a problem)
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Algebra Tiles
- Grid paper
- Math blocks
- Area models, such as rectangles filled with base-ten pieces
- Flash cards
- Pictorial representations
- Teacher-generated worksheets

List of Technology Resources:

- Khan Academy— <http://khanacademy.org>
- Cool Math – www.coolmath.com/
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

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Expressions and Equations (EE)

3.EE.1 / 3.EE.2 / 3.EE.3 / 3.EE.4 / 3.EE.5 / 3.EE.6 / 3.EE.7 / 3.EE.8 / 3.EE.9

Essential Understandings:

- Properties of operations are used to determine if expressions are equivalent.
- Substitution is used to determine whether a given number in a set makes an equation or inequality true.
- Variables may be used to represent a specific number, or, in some situations, to represent all numbers in a specified set.
- When one expression has a different value than a related expression, an inequality provides a way to show that relationship between the expressions: the value of one expression is greater than (or greater than or equal to) the value of the other expression instead of being equal.
- Solutions of inequalities can be represented on a number line.
- Variables in algebraic equations can be expressed in graphs to represent numbers and generalize mathematical problems in real-world situations.

Essential Questions:

- How do we determine if a variable is independent or dependent in an expression or equation?
- How are variables defined and used?
- How does the structure of equations and/or inequalities help us solve equations and/or inequalities?
- How does the substitution process help in solving problems?
- Why are variables used in equations?
- What might a variable represent in a given situation?
- How are inequalities represented and solved?
- How can the order of operations be applied to evaluating expressions, and solving from one-step to multi-step equations?
- What are some possible real-life situations to which there may be more than one solution?
- How do we express a relationship mathematically?
- How do we determine the value of an unknown quantity?

Student will be able to:

- Apply and extend previous knowledge of arithmetic to write and evaluate numerical expression with whole-number exponents.
- Demonstrate understanding of algebraic expressions by writing, reading, and evaluating expressions with variables (e.g., express the calculation “subtract y from 5” as $5 - y$).
- Demonstrate understanding of algebraic expressions involving variables by identifying parts of an expression using mathematical terms, viewing one or more parts of an expression as a single entity.
- Demonstrate understanding of algebraic expressions involving variables by evaluating expressions at specific values for their variables, including expressions that arise from formulas in real-world problems.
- Demonstrate understanding of algebraic expressions involving variables by performing arithmetic operations, including those involving whole-number

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exponents, in conventional order when there are no parentheses to specify a particular order (order of operations).

- Generate equivalent expressions by applying properties of operations.
- Compare two expressions and determine if they are equivalent (e.g., $y + y + y = 3y$).
- Solve equations and inequalities by answering the question, “Which values from a specified set make this true?” and by using substitution to determine if an equation or inequality is true.
- Utilize variables to write expressions when solving real-world and mathematical problems, understanding that variables represent an unknown number or number in a specified set.
- Solve equations in real-world and mathematical contexts of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all non-negative rational numbers.
- Represent a constraint in a real-world or mathematical problem by writing an inequality in the form $x < a$ or $x > a$, recognize that inequalities in such a form have an infinite number of solutions, and graph solutions for such inequalities.
- Write, read, and evaluate expressions involving variables by representing and simplifying quantitative relationships between dependent and independent variables, analyze the relationship between the dependent and independent variables using graphs and tables, and relate findings to the equation.

Performance Tasks:

- Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:
- Teacher-generated assignments and quizzes such as use substitution to determine monthly car payments based on a four-year plan (See class activities)
- Text: Steck-Vaughn and Contemporary’s Unit Reviews and/or exercises as quizzes and chapter tests.
- Chapter/unit reviews/tests from core texts such as New Readers Press Math Sense

Other Evidence

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

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Sample Classroom Activities and/or Lesson Plans:

- Use pictures to write a numerical expression
- Match word cards to parts of a numerical expression
- Use counters and an empty box to represent a variable, create an equation
- Practice solving equations and simplifying expressions
- Estimate weekly wages based on commission
- Use substitution to determine monthly car payments based on a three-year plan
- Follow one shipping company's business for one day. Track the time and distance using $d=rt$.

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology: Study Stack, websites as assigned
- One-to-one intervention
- Peer tutoring
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Think out loud (demonstrate how to think about a problem)
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Calculators
- Newspaper articles and advertisements
- Multiplication boxes
- Counters
- Fictional budgets and financial records

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Kuta Software— <https://www.kutasoftware.com>
- CK-12 - <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

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NUMBER AND OPERATIONS IN BASE TEN (NBT)

3.NBT.1 / 3.NBT.2 / 3.NBT.3 / 3.NBT.4 / 3.NBT.5 / 3.NBT.6 / 3.NBT.7 / 3.NBT.8 /
3.NBT.9 / 3.NBT.10 / 3.NBT.11 / 3.NBT.12 / 3.NBT.13 / 3.NBT.14 / 3.NBT.15

Essential Understandings:

- The place value of whole and decimal numbers is based on groups of ten and the value of a number is determined by the place of its digits.
- The standard algorithm for addition and subtraction relies on adding or subtracting like base-ten units.
- Whole numbers are read from left to right using the name of the period; commas are used to separate periods.
- A whole or decimal number can be written using its name, standard, or expanded form and can be compared to other whole or decimal numbers using greater than, less than or equal to symbols.
- Flexible methods of computation involve grouping numbers in strategic ways.
- Multiplication and division are inverse operations.
- The four operations are interconnected.
- In a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
- Multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. The exponent not only indicates how many places the decimal point is moving but also that you are multiplying or making the number 10 times greater, three times when you multiply by 10^3 (e.g. $3.4 \times 10^3 = 3.4 \times (10 \times 10 \times 10) = 3.4 \times 1,000 = 3,400$.)

Essential Questions:

- How does the position of a digit in a number affect its value, and how can the value of digits be used to compare two numbers?
- In what ways can numbers be composed and decomposed?
- How are the four basic operations related to one another?
- How does understanding place value help you solve multi-digit addition and subtraction problems and how can rounding be used to estimate answers to problems?
- What occurs when whole numbers and decimals are multiplied by 10 or powers of 10?
- Using less than, greater than, or equal to symbols, how can whole and decimal numbers (with like or unlike forms) be compared?

Student will be able to:

(What does mastery look like?)

- Classify place values in a multi-digit whole number (e.g., identify multiples of ten in place value).
- Interpret and express multi-digit whole numbers using base-tens, mathematical names, and expanded notation, and identify and use appropriate math symbols such as $<$, $>$, $=$ to compare two multi-digit numbers.
- Round multi-digit whole numbers to any place.
- Compute addition and subtraction problems with multi-digit numbers using the standard algorithm.
- Use place value understanding and properties of operations to multiply a whole number of four or fewer digits by a one-digit number and to multiply two two-digit

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numbers, using area models, rectangular arrays, or equations to illustrate and explain the calculation.

- Use place value understanding, properties of operations, and/or the relationship between multiplication and division to find quotients and remainders with up to four digit dividends and one-digit divisors, using area models, rectangular arrays, or equations to illustrate and explain the calculation.
- Demonstrate understanding of place value by recognizing and indicating that a number in one place is ten times greater than it would represent in the place to the right or that the same number is one tenth of the value of place on the left.
- Determine patterns in the number of zeroes in a product as well as the placement of decimals when multiplying or dividing by a power of ten.
- Identify, read, write, and compare decimals to the thousandths.
- Identify, read, and write decimals to thousandths using appropriate base-ten numerals, math names, and expanded forms.
- Use the symbols $<$, $>$, and $=$ to compare two decimals to thousandths and appropriately express place value.
- Round decimals to any place applying place value understanding.
- Multiply multi-digit whole numbers using the standard algorithm.
- Use strategies based on place value, properties of operations, and the relationship between multiplication and division to find whole number quotients and remainders with up to four-digit dividends and two-digit divisors, using area models, rectangular arrays, or equations to illustrate and explain the calculation.
- Using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction, add, subtract, multiply, and divide decimals to the hundredths and relate the strategies to a written method to explain the reasoning.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Quiz: Match number quantities to names/quantities to written numerals/names to written numbers
- Use teacher made quizzes and tests
- Steck Vaughn and Contemporary's Test Preparation Unit Reviews and/or exercises as quizzes and chapter tests (see below)
- Math Journals-graded
- Facilitate group work/monitoring conversations during group work

Other Evidence:

- Outside assignments
- Facilitate group work/monitoring conversations during group work
- Facilitate individual monitoring by instructor during independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Create math grids with plan paper to represent, ones, tenths, hundreds, and thousandths
- Use math games to find products and quotients of whole numbers

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- Use “The Frayer Model” to organize concepts of place value and base-ten numbers
- Using number pattern identifications to expand knowledge of number sense, for example, counting by multiples to understand multiplication concepts
- Using money to learn about place value and trading coins
- Use three and four digits for addition and subtraction
- Use three and four digits for multiplication and division

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutoring
interactive technology
- Manipulatives
- Peer teaching
- Procedures with different objects and concepts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Place value charts
- Arrow cards
- Rectangular arrays
- Area models
- Possible Texts:
 - Number Concepts (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement (2014). New Readers Press/Pre-High School Equivalency
 - Achieving TABE Success in Mathematics Level M, (2006). McGraw Hill

List of Technology Resources:

- Khan Academy— <https://www.khanacademy.org/>
- Cool Math--- www.coolmath.com
- Greg Tang Math— www.gregtangmath.com
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

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NUMBER AND OPERATIONS - FRACTIONS (NF)

3.NF.1 / 3.NF.2 / 3.NF.3 / 3.NF.4 / 3.NF.5 / 3.NF.6 / 3.NF.7 / 3.NF.8 / 3.NF.9 / 3.NF.10 /
3.NF.11 / 3.NF.12 / 3.NF.13 / 3.NF.14

Essential Understandings:

- Fractions can be represented visually and in written form.
- Comparisons are valid only when the fractions or decimal numbers refer to the same whole.
- Fractions and mixed numbers are composed of unit fractions and can be decomposed as a sum of unit fractions.
- Improper fractions and mixed numbers can represent the same value.
- Addition and subtraction of fractions involves joining and separating parts referring to the same whole.
- A product of a fraction times a whole number can be written as a multiple of a unit fraction.
- Fractions with denominators of 10 can be expressed as an equivalent fraction with a denominator of 100.
- Fractions with denominators of 10 and 100 may be expressed using decimal notation.
- Benchmark fractions and other strategies aid in estimating the reasonableness of results of operations with fractions.
- The use of area models, fraction strips, and number lines, are effective strategies to model sums, differences, products, and quotients.
- Equivalent fractions are critical when adding and subtracting fractions with unlike denominators.
- Fractions are division models.
- Multiplication can be interpreted as scaling/resizing (multiplying a given number by a fraction greater than 1 result in a product greater than the given number and multiplying a given number by a fraction less than 1 result in a product smaller than the given number).
- The knowledge of fractions and equivalence of fractions can be used to develop algorithms for adding, subtracting, multiplying, and dividing fractions.

Essential Questions:

- How are fractions used in problem-solving situations?
- How are fractions composed, decomposed, compared and represented?
- Why is it important to identify, label, and compare fractions as representations of equal parts of a whole or of a set?
- How can multiplying a whole number by a fraction be displayed as repeated addition (as a multiple of a unit fraction)?
- How can visual models be used to determine and compare equivalent fractions and decimals?
- How can decimals through the hundredths place be compared and ordered?
- What is a reasonable estimate for a solution (answers)?
- How do operations with fractions relate to operations with whole numbers?
- What do equivalent fractions represent and why are they useful when solving equations with fractions?
- What models or pictures could aid in understanding a mathematical or real-world problem and the relationships among the quantities?
- When can model(s) or picture(s) be used to solve a mathematical or real-world problem?

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- What are the effects of multiplying by quantities greater than one compared to the effects of multiplying by quantities less than one?

Student will be able to:

(What does mastery look like?)

- Explain, compare, and generate equivalent fractions by using visual fraction models.
- Compare and contrast two fractions with different numerators and different denominators using appropriate math symbols and fraction models, understanding that comparisons are only made if fractions refer to the same whole.
- Explain the addition and subtraction of fractions as joining and separating parts of the same whole.
- Dissect a fraction into a sum of fractions with the same denominator recording each as an equation, explaining the decomposition with a visual model.
- Demonstrate understanding of fractions by adding and subtracting mixed numbers with the same denominators.
- Solve word problems involving addition and subtraction of fractions with the same denominators.
- Utilize and extend previous understanding of multiplication by recognizing that $5/4$ is the equivalent of the whole number 5 multiplied by the fraction $1/4$ or that a over b (a/b) is a multiple of 1 over b ($1/b$).
- Utilize previous understanding of multiplication to multiply a fraction by a whole number, using a fraction model such as $3 \times (2/5) = 6 \times (1/5)$ which is a multiple of $1/b$.
- Apply and extend comprehension of multiplication to solving word problems which involve the multiplication of a fraction and a whole number.
- State a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100, and utilize this skill to add two fractions with denominators of 10 and 100.
- Apply decimal notation for fractions with denominators of 10 and 100.
- Compare two decimals to the hundredths place by analyzing size using appropriate math symbols and rules of valid comparison.
- Apply the strategy of equivalent fractions to add and subtract fractions with unlike denominators, including mixed numbers, producing equivalent sums or differences with like denominators.
- Apply the strategy of using benchmark fractions and number sense of fractions to solve word problems that include addition and subtraction of fractions referring to the same whole and including cases of unlike denominators, using mental estimation to assess reasonableness of answers.
- Explain that a fraction is a division of its numerator by its denominator and apply this understanding to solve word problems which include division of whole numbers resulting in fraction or mixed number forms.
- Apply and extend previous knowledge of multiplication to multiply a fraction or whole number by a fraction and explain it as a result of a sequence of operations (e.g., $(a/b) \times q$ can be explained as $a \times q / b$).
- Apply and extend previous knowledge of multiplication to find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths or by showing that the area could similarly be found by multiplying the side lengths.
- Interpret multiplication as scaling by comparing size of product to size of each factor using estimation.

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- Interpret multiplication as scaling by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number while multiplying a given number by a fraction less than 1 results in a product less than the given number.
- Use visual models or equations to compute real world problems involving multiplication of fractions and mixed numbers.
- Apply and extend previous knowledge of division by interpreting division of a unit fraction by a non-zero whole number and computing quotients using the relationship between multiplication and division to explain the process.
- Interpret division of a whole number by a unit fraction and compute quotients using the relationship between multiplication and division to explain the process.
- Demonstrate previous comprehension of division by solving real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Steck Vaughn and Contemporary's Unit Reviews and/or exercises as quizzes and chapter tests (see below)
- Use teacher made quizzes and tests
- Graded Journal entries

Other Evidence:

- Use "The Frayer Model" to deepen the understanding process
- Observing math journal entries
- One on one observations of independent work
- Facilitate group work and monitor conversations during group work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Re-create a recipe to accommodate double portions and half size portions
- Compare different measurement options, such as cups and quarts, etc.
- Divide cookies in different sizes and determine equivalent portions
- Have students work in groups of three on solving common unit fraction word problems
- Have students do classroom surveys of types of pets that live in their households and record their findings in fraction form
- Compare metric measurements with traditional measurements
- Compare interest rates

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Math manipulatives
- Flash cards
- One-to-one tutoring by instructor

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Peer tutoring
- Interactive Technology
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Number lines
- Flash cards
- Fraction game cards
- Dice
- Rectangular arrays
- Area models
- Grid paper
- Math blocks
- Possible Texts:
 - Concepts (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement (2014). New Readers Press/Pre-High School Equivalency
 - Achieving TABE Success in Mathematics Level M, (2006). McGraw Hill

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Cool Math— www.coolmath4kids.com/fractions
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

The NUMBER SYTEM (NS)

3.NS.1 / 3.NS.2

Essential Understandings:

- The relationship of the location of the digits and the value of the digits is part of understanding multi-digit operations.
- Various operations can be performed and represented using multiple formats (manipulatives, diagrams, real-life situations, equations).

Essential Questions

- What role does place value play in multi-digit operations?

Student will be able to:

- Divide multi-digit numbers using the standard algorithm.
- Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Steck Vaughn and Contemporary's Unit Reviews and/or exercises as quizzes and chapter tests (see below)

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Use teacher made quizzes and tests
- Graded Journal entries

Other Evidence:

- Use “The Frayer Model” to deepen the understanding process
- Observing math journal entries
- One on one observations of independent work
- Facilitate group work and monitor conversations during group work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students work in groups of four to reduce a recipe and record new measurements
- Write a real-life math problem involving hourly wages with weekly pay
- Have students use a four-function calculator to check answers
- Use integers to keep track of scores in an interactive football game
- Analyze temperatures over the last four years. Students work in groups choosing different areas from any continent
- Have students re-write recipes to increase number of servings
- Have each student make up Scientific Cards in two different colors, one color for standard decimal notation and the other for scientific notation

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Math manipulatives
- Flash cards
- One-to-one tutoring by instructor
- Peer tutoring
- Interactive Technology
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Number lines
- Flash cards
- Fraction game cards
- Dice
- Rectangular arrays
- Area models
- Grid paper
- Math blocks
- Possible Texts:
 - Concepts (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement (2014). New Readers Press/Pre-High School Equivalency
 - Achieving TABE Success in Mathematics Level M, (2006). McGraw Hill

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Cool Math— www.coolmath4kids.com/fractions
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)

3.RP.1 / 3.RP.2

Essential Understandings:

- A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percent.
- A ratio or a rate expresses the relationship between two quantities. Ratio and rate language are used to describe a relationship between two quantities (including unit rates).
- A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.

Essential Questions:

- How are ratios and rates similar and different?
- What is the connection between a ratio/rate and a fraction?
- How do rates, ratios, percentages and proportional relationships apply to our world?
- When and why is it appropriate to use proportional comparisons?
- How does comparing quantities describe the relationship between them?

Student will be able to:

- Demonstrate the concept of a ratio by using ratio language to describe a ratio relationship between two quantities.
- Demonstrate the concept of a unit rate a/b associated with a ratio $a:b$ (with b not equal to zero) by using ratio language to show ratio relationships.

Evidence for Assessing Learning

Performance Tasks:

- Teacher made assignments and quizzes: on rates, ratios, percentages and proportional relationships, including: solving multi-step ratio and percent problems such as interest, tax, discount, unit price etc. (See class activities)

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

Possible Texts:

- Ratios, Proportions and Percents (2016). McGraw Hill/College and Career Readiness Practice Workbook.
- Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement (2014). New Readers Press/Pre-High School Equivalence

Other Evidence:

- Monitor math journal entries
- Facilitate and monitor group work
- Facilitate and monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson plans:

- Have students bring in different cookie, cereal boxes, juice containers, or other food items to calculate price per unit.
- Calculate difference in gas used, (per gallon), going at a constant rate up a hill verse going at the same rate over a flat mile.
- All students should record their findings of math activities in their math journals.
- Use a walking field trip to estimate time in minutes based on distance and speed. Have students work in groups of four, as some may walk slower or faster.
- Learning Activities: (Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)
- Interactive technology as assigned by instructor to support instruction interactive websites/illustrations, Study Stack – vocabulary support, etc. (See below)
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Practice work sheets-teacher made
- Ads for gasoline per gallon
- Various items from the grocery store

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Calculators

Possible Texts:

- *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
- *EMPower Math, Keeping Things in Proportion: Reasoning with Ratios. Student Edition.* (2011). Contemporary/McGraw-Hill.
- *Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement* (2014). New Readers Press/Pre-High School Equivalency
- *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill

List of Technology Resources:

- Kahn Academy- <https://www.khanacademy.org/>
- Teaching Ideas— www.teachingideas.co.uk
- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

MEASUREMENT AND DATA (MD)

3. MD.1 / 3.MD.2 / 3.MD.3 / 3.MD.4 / 3.MD.5 / 3.MD.6 / 3.MD.7 / 3.MD.8 / 3.MD.9 /
3.MD.10 / 3.MD.11 / 3.MD.12

Essential Understandings:

- Converting from larger to smaller units of measurement in the metric system is done by multiplying by powers of ten.
- Perimeter is a real-life application of addition and subtraction.
- Area is a real-life application of multiplication and division.
- When converting measurements within one system, the size, length, mass, volume of the object remains the same.
- Measurement problems can be solved by using appropriate tools.
- Volume of three-dimensional figures is measured in cubic units.
- Volume is additive and/or it is the multiplication of three dimensions (length, width and height).
- Multiple rectangular prisms can have the same volume.
- Volume can be used to solve a variety of real-life problems.
- The concepts of distances, intervals of time, volume, masses of objects, and money can be expressed as measurements of a larger unit in terms of a smaller unit.
- Angles are measured in the context of a central angle of a circle.
- Angles are composed of smaller angles.

Essential Questions:

- How are the units of measure within the metric system related?
- How do you find the area and perimeter of geometric figures and how can using the formulas for perimeter and area help you solve real-world problems?
- Why does the size, length, mass, volume of an object remain the same when converted to another unit of measurement?
- What is volume and how is it used in real life?

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- How does the area of rectangles relate to the volume of rectangular prisms?
- What are the types of angles and the relationships?
- How are angles applied in the context of a circle?
- How are protractors used to measure and aid in drawing angles and triangles?
- How can an addition or subtraction equation be used to solve a missing angle measure when the whole angle has been divided into two angles and only one measurement is given?

Student will be able to:

(What does mastery look like?)

- Name sizes of measurement units including km, m, cm, kg, g, lb., oz., L, ml, hr., min, sec., and express measurement equivalents from larger to smaller units within a single measurement system using a two-column table.
- Solve word problems involving measurements and conversions of measurements using the four operations, simple fractions, decimals, number lines, or diagrams to answer problems involving distance, intervals of time, liquid volumes, mass, and money, showing representative quantities.
- Utilize area and perimeter formulas for rectangles to solve real world and mathematical problems.
- Construct a line plot of a data set of measurements in fraction units, and solve problems of addition and subtraction with fractions involving information presented in the line plots.
- Display a set of data by making line plots for measurements, using operations on fractions for this grade to solve problems involving information presented in the line plots.
- Demonstrate understanding of angle concepts by identifying angles as geometric shapes formed wherever two rays share a common endpoint.
- Recognize concepts of angle measurement by understanding that for a circle with its center the common endpoint of two rays, an angle is measured as the fraction of the circular arc between the points where the opposite ends of the rays intercept the circle, and an angle measuring $\frac{1}{360}$ th of a circle is a one-degree angle and a common unit of measurement.
- Recognize concepts of angle measurement by identifying angles that turn through in one-degree angles as having an angle measure of “n” degrees.
- Demonstrate understanding of angle concepts by using a protractor to measure angles in whole-number degrees and to sketch angles of a designated measure.
- Find measurement of unknown angles utilizing prior knowledge of angle measurement as an additive in which addition /subtraction operations can be used to solve real world and mathematical problems, as in finding unknown angles on a diagram.
- Solve real world multi-step problems involving conversion among different-sized standard measurements units within a given measurement system.
- Identify volume as an attribute of solid figures and relate concepts of volume measurement to cubes with side length 1 unit, called a “unit cube,” and used to measure volume.
- Identify volume as an attribute of solid figures and relate concepts of volume measurement to solid figures that can be packed without gaps using “n” unit cubes to determine volume for a solid figure as measuring a quantity of “n” cubic units.
- Count unit cubes to measuring volume by counting unit cubes using cubic cm, cubic in, cubic ft, and other units.

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Apply the operations of multiplication and addition to solve real world and mathematical problems involving volume, by finding the volume of a right rectangular prism by packing it with unit cubes and showing that this value can be found by multiplying the height by area of the base.
- Apply the operations of multiplication and addition to solve real world and mathematical problems involving volume by applying the formulas $V=lwh$ and $V=bh$ to rectangular prisms.
- Apply the operations of multiplication and addition to solve real world and mathematical problems involving volume by relating volume to operations of addition to find volumes of solid figures composed of two non-overlapping right rectangular prisms and adding the volumes of the non-overlapping parts.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Create a graphic representation of level appropriate quantities
- Contemporary's Unit Reviews and/or exercises as quizzes and chapter tests. (see below)
- Teacher made quizzes and tests
- Math journals-graded entries
- Facilitated group work/ monitoring conversations and work with rubric

Other Evidence:

- Use "The Frayer Model" to enhance concepts
- Observation of facilitated independent work
- Facilitated group work/ monitoring conversations and work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Display various measurement tools in different units
- Have student measure different objects in the classroom using various measurement tools
- Record measurement on chart paper
- Introduce math manipulatives that measure mass and volume
- Demonstrate measurement of area vs. volume vs. perimeter of a specific object
- Students will work in groups on math worksheets involving measurement
- Create line plots that display measurements in fraction units
- Introduce geometric shapes as math manipulatives

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutor
- Peer teaching
- Use different objects and put material in different context-use different measuring tools
- Interactive technology- websites as assigned (see below)

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers in standard measurement units
- Protractors
- String
- Straws
- Geometric math boards with rubber bands
- Graph paper
- Math journal
- Geometric math blocks
- Scales
- Possible Texts:
 - EMPower Math – Over, Around, and Within: Geometry and Measurement. (2012). Contemporary.
 - Math 2 workbook: Algebraic Thinking, Data Analysis, and Probability (2014). New Readers Press

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Cool Math— www.coolmath.com
- Kuta Software— <https://www.kutasoftware.com>
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

GEOMETRY (G)

3. G.1 / 3.G.2 / 3.G.3 / 3.G.4 / 3.G.5 / 3.G.6 / 3.G.7 / 3.G.8 / 3.G.9 / 3.G.10 / 3.G.11

Essential Understandings:

- Shapes can be classified by properties (or attributes) of their lines and angles.
- Angles are measured in the context of a central angle of a circle.
- Angles are composed of smaller angles.
- Two-dimensional geometric figures are composed of various parts that are described with precise vocabulary and can be classified based upon their properties (attributes).
- In a coordinate plane, the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in the direction of the second axis.
- The coordinate plane can be used to model and compare numerical patterns.
- Figures that can be folded on a center line to produce two matching parts are symmetrical.
- The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems.
- Graphing objects in a four quadrant plane can provide ways to measure distances and identify that shapes have specific properties.
- Volume of a rectangular prism can be determined by multiplying the length, width and height dimensions when the dimensions are fractional lengths.
- Two- and three-dimensional shapes and spaces are defined by their properties; real world and geometric structures are composed of these shapes and spaces.

Essential Questions:

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Why is it important to use precise language and mathematical tools in the study of two-dimensional and three-dimensional figures?
- How can describing, classifying and comparing attributes of two-dimensional shapes (nets) be useful in solving problems in our three-dimensional (dot paper drawings) world?
- Problems of area of polygons can be solved by composing and decomposing the polygons.
- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- How are parallel lines and perpendicular lines used in classifying two-dimensional shapes?
- What are the types of angles and the relationships?
- How are angles applied in the context of a circle?
- How are protractors used to measure and aid in drawing angles and triangles?
- Why is it important to use precise language and mathematical tools in the study of two-dimensional figures?
- How can describing, classifying and comparing properties of two-dimensional shapes be useful in solving real-world problems?
- How can an addition or subtraction equation be used to solve a missing angle measure when the whole angle has been divided into two angles and only one measurement is given?
- What is the purpose of a coordinate plane?
- How can graphing points on the coordinate plane help to solve real world and mathematical problems?
- How can the line of symmetry be identified and drawn in a two-dimensional figure?

Student will be able to:

(What does mastery look like?)

- Plot points and draw line segments, rays, angles, perpendicular lines, and parallel lines, identifying these in two-dimensional figures.
- Categorize two-dimensional figures based on the presence/absence of parallel or perpendicular lines or the presence/absence of angles of a specified size, and identify and categorize right triangles.
- Identify a line of symmetry as a line running across a two-dimensional figure so that the figure can be folded along the line into two matching parts, and identify and draw lines of symmetry.
- Define a coordinate plane, identify the x-axis and y-axis, and demonstrate comprehension of ordered pairs as directional distance on the x and y axis.
- Show a real life or mathematical problem and illustrate it graphically by plotting points in the first quadrant of a coordinate plane, interpreting coordinate values in context.
- Demonstrate comprehension that attributes belonging to a category of two-dimensional figures apply to all subcategories (e.g., all rectangles have four right angles; squares are rectangles; therefore, squares have four right angles).
- Sort and classify two-dimensional figures based on their properties and characteristics.
- Solve real-world and mathematical problems by finding the area of triangles, right triangles, polygons, and quadrilaterals by composing into rectangles and decomposing into triangles and other shapes.

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Solve real-world and mathematical problems involving volume for right rectangular prisms with fractional edge lengths by using modeling with unit cubes, approximating the fractional measure and applying the formula $V=lwh$ and $V=bh$ to rectangular prisms with fractional edge lengths. Solve real-world and mathematical problems involving area by using ordered pairs, sketching a polygon, and finding the length of each side in the coordinate plane.
- Solve real-world and mathematical problems involving surface area by representing three-dimensional figures using nets of rectangles and triangles.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher made assignments and quizzes: for example, given data construct a line plot
- Text: Steck-Vaughn and Contemporary's Test Preparation Unit Reviews and/or exercises as quizzes and chapter tests (see below)
- Assign outside projects that involve measurement and data entry
- Monitor independent assignments

Other Evidence:

- Math journal entries
- Facilitate group work/ monitoring conversations and work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Test for line symmetry using the folding method:
<https://www.mathsisfun.com/activity/shapes-symmetry.html>
- Have a class discussion about lines and angles.
- Identify & take pictures of lines and angles in the classroom, building, and at home.
- Construct a reference poster depicting various lines and angles.
- Construct a reference poster depicting area, surface area, and volume methods and attributes.
- Have student sketch a coordinate plane and label each axis.
- Provide blank coordinate planes and have students label them.
- Plot several points and connect them to reveal a shape or letter.
- Define and classify two-dimensional figures, name their similar **characteristic**.

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutor
- Interactive technology
- Peer teaching
- Use different objects and put material in different context.
- Work with groups that are experiencing difficulties with measurement data

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards, and games such as Battleship)

List of Instructional Materials:

- Graph paper
- Colored pencils
- Rulers
- Protractors
- Math journal
- Different types of real-world math problems drawn from everyday life or job related
- Possible Texts:
 - EMPower Math – Over, Around, and Within: Geometry and Measurement. (2012). Contemporary.
 - Math Sense 2: Algebraic Thinking, Data Analysis, and Probability (2014). New Readers Press
 - New Readers Press Math Sense 1 and 2
 - McGraw Hill Common Core Basics Mathematics

List of Technology Resources:

- i-Pathways Basic Math — www.i-pathways.org
- Khan Academy— <http://khanacademy.org/>
- Cool Math— www.coolmath.com
- Symmetry Matching Game - <https://www.topmarks.co.uk/symmetry/symmetry-matching>
- Kuta Software— <https://www.kutasoftware.com/freeige.html>
- Ck-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>

Statistics and Probability (SP)

3.SP.1 / 3.SP.2 / 3.SP.3 / 3.SP.4

Essential Understandings

- Statistical questions and the answers account for variability in a data set.
- The distribution of a data set is described by its center, spread, and overall shape.
- Measures of central tendency for a numerical set of data are summaries of the values using a single number.
- Measures of variability describe the variation of the values in the data set using a single number.
- Different representations (written descriptions, tables, scatter plots, histograms, box and whisker plots, graphs, and equations) of the relationships between varying quantities may have different strengths and weaknesses.

Essential Questions

- What is the value of using different data representations?
- When is one data display better than another? How can data be displayed strategically?
- What makes a good statistical question?

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

Student Will be able to:

- Develop understanding of statistical variability by applying a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
- Develop understanding of statistical variability by demonstrating that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- Develop understanding of statistical variability by demonstrating the measure of center for a numerical data set summarizing all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Summarize and describe distributions by displaying numerical data in plots on a number line, including dot plots, histograms, and box plots.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher made assignments and quizzes (See class activities)
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson plans:

- Take a survey of each student's age and birthdate in one class
- Explain how a statistical question anticipates variability in data related to the question and accounts for it in the answer
- Assign groups to create a reference poster of the different ways to display numerical data.
- Have students work in groups to collect data on a topic of each group's choice. Each group should display numerical data in a different way, for example: one group could plot on a number line, another group on dot plots, and still another using box and whisker
- Adding to this classroom activity, students must explain in their math journals the distribution on the data collected.
- All activities should be recorded on tables, tree diagrams, and on organized lists

NRS Math Level 3

Low-Intermediate Basic Education (Grade Levels 4-5)

Learning Activities: (Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One on one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Chart paper
- Interactive technology
- Dice, coins, cards, rulers
- Markers and colored pencils
- Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook. (2013). Steck-Vaughn. Common Core Basics: Mathematics. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- Cool Math— www.coolmath.com
- Math Planet— www.mathplanet.com/
- Math Antics www.mathantics.com

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 4**

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NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)

4.RP.1 / 4.RP.2 / 4.RP.3 / 4.RP.4

Essential Understandings:

- A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percent.
- Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.).
- Rates, ratios, percentages and proportional relationships express how quantities change in relationship to each other and can be represented in multiple ways.
- Rates, ratios, percentages and proportional relationships can be applied to multi-step ratio and percent problems along with other problem solving situations such as interest, tax, discount, etc.

Essential Questions:

- When is it useful to be able to relate one quantity to another?
- How can models illustrate proportional relationships?
- How can proportional relationships be used to solve ratio and percent problems?
- How can scale drawings be used to compute actual lengths and area?

Student will be able to:

(what does mastery look like)

- Solve real world and mathematical problems using ratio and rate reasoning by making tables of equivalent ratios connecting quantities with whole number measurements, finding missing values for each table of coordinate pairs, and plotting them on a coordinate plane.
- Use ratio and rate reasoning to solve unit rate problems that include unit price and a constant speed.
- Use ratio and rate reasoning to find a percent of a quantity as a rate per 100 and to find the whole given a part and the percent.
- Use ratio and rate reasoning to convert measurement units, manipulate units and transform units while multiplying or dividing quantities.
- Calculate unit rates associated with ratios of fractions, lengths, and areas in like or different units.
- Identify and show proportional relationships between quantities by determining whether two quantities are in a proportional relationship.
- Identify and show proportional relationships between quantities by identifying the proportional constant in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- Identify and show proportional relationships between quantities by solving equations.

NRS Level 4 Math

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- Identify and show proportional relationships between quantities by explaining in writing what a point (X,Y) on a graph of a proportional relationship means in terms of a situation, paying particular attention to the points (0,0) and (1,r) where r is the unit rate.
- Solve multistep ratio and percent problems by using proportional relationships.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher made assignments and quizzes: on rates, ratios, percentages and proportional relationships, including: solving multi-step ratio and percent problems such as interest, tax, discount, unit price etc. (See class activities)
- Possible Texts:
 - *Ratios, Proportions and Percents* (2016). McGraw Hill/College and Career Readiness Practice Workbook.
 - *Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement* (2014). New Readers Press/Pre-High School Equivalency

Other Evidence:

- Monitor math journal entries
- Facilitate and monitor group work
- Facilitate and monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students bring in different cookie, cereal boxes, juice containers, or other food items to calculate price per unit.
- Calculate difference in gas used, (per gallon), going at a constant rate up a hill verse going at the same rate over a flat mile.
- Have students survey online shopping to calculate the final cost of sale items of their choice, including sales tax.
- All students should record their findings of math activities in their math journals.
- Use a walking field trip to estimate time in minutes based on distance and speed. Have students work in groups of four, as some may walk slower or faster.

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Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction interactive websites/illustrations, Study Stack – vocabulary support, etc. (See below)
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Practice work sheets-teacher made
- Ads for gasoline per gallon
- Various items from the grocery store
- Calculators
- *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
- *EMPower Math, Keeping Things in Proportion: Reasoning with Ratios. Student Edition.* (2011). Contemporary/McGraw-Hill.
- *Possible Texts:*
 - *Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement* (2014). New Readers Press/Pre-High School Equivalency
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kahn Academy- <https://www.khanacademy.org/>
- Teaching Ideas— www.teachingideas.co.uk
- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- Cool Math— www.coolmath.com
- Math Planet— www.mathplanet.com/
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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THE NUMBER SYSTEM (NS)

4.NS.1 / 4.NS.2 / 4.NS.3 / 4.NS.4 / 4.NS.5 / 4.NS.6 / 4.NS.7 / 4.NS.8 / 4.NS.9

Essential Understandings:

- Rational numbers use the same attributes as whole numbers.
- The quotative (making groups of a certain size) and partitive (sharing equally or dealing out) types of division and measurement are applied to numbers within the real number system (fractions, decimals, integers and rational and irrational numbers).
- Quantities having more or less than zero are described using positive and negative numbers.
- Number lines are visual models used to represent the density principle: between any two whole numbers are many rational numbers, including decimals and fractions.
- The rational numbers can extend to the left or to the right on the number line, with negative numbers going to the left of zero, and positive numbers going to the right of zero.
- The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems.
- Graphing objects in a four-quadrant graph can provide ways to measure distances
- Rational numbers can be represented with visuals (including distance models), language, and real-life contexts.
- There are precise terms and sequences to describe operations with rational numbers.
- Every rational number has a decimal expansion.
- Properties of operations with whole and rational numbers also apply to all real numbers.
- Absolute value is a number's distance from zero (e.g., $|-3| = 3$.)

Essential Questions:

- How are various operations (addition, subtraction, multiplication and division) represented, interpreted, and related to realistic situations and to other operations?
- How are positive and negative numbers used?
- How do rational numbers relate to integers?
- What can be modeled on the coordinate plane?
- Why are quantities represented in multiple ways?
- How can quantities be represented and what is the rationale for selecting a specific representation?
- What does the absolute value of a number represent?

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Student will be able to:

(what does mastery look like)

- Demonstrate understanding of fractions by analyzing and solving quotients of fractions and solving word problems involving division of a fraction by a fraction.
- Show that positive and negative numbers represent quantities in real-world contexts and can be used together to describe quantities of opposite directions or values.
- Show a rational number as a point on the number line.
- Use previous knowledge of rational numbers to extend number line diagrams and coordinate axes to include negative numbers.
- Identify that the opposite of the opposite of a number is itself and identify that numbers on a number line with opposite signs are on opposite sides of 0.
- Recognize and plot the location of ordered pairs containing signed numbers in quadrants on coordinate planes, and show that the position of points of two ordered pairs differing only by sign are related by reflections across one or both axes.
- Indicate and plot the position of integers and other whole numbers on vertical or horizontal line diagrams and indicate and plot ordered pairs on a coordinate plane.
- Demonstrate understanding of ordering and absolute value of rational numbers by interpreting expressions of inequality of two numbers in terms of their relative positions on a number line diagram.
- Demonstrate understanding of ordering and absolute value of rational numbers by expressing statements of order for rational numbers in real world situations.
- Demonstrate understanding of ordering and absolute value of rational numbers by recognizing that the distance from zero of a rational number is the absolute value of the number and interpreting absolute value in real world situations as magnitude for a positive or negative quantity.
- Demonstrate understanding of ordering and absolute value of rational numbers by distinguishing between comparisons of absolute value from statements of order of rational numbers.
- Graph points in all four quadrants of the coordinate plane using prior knowledge of coordinates and absolute value to find the distance of points with the same first or second coordinate.
- Use prior knowledge of operations to add and subtract rational numbers and graph results on a line diagram.
- Apply and extend previous understanding of addition and subtraction of rational numbers by describing situations in which opposite quantities combine to make 0.
- Apply and extend previous understanding of addition and subtraction of rational numbers by showing understanding of $p + q$ as the number

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located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative.

- Demonstrate that a number and its opposite are additive inverses, so their sum is 0, and interpret sums of rational numbers in real-world contexts.
- Apply and extend previous understanding of addition and subtraction of rational numbers by demonstrating the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- Demonstrate previous knowledge of addition and subtraction of rational numbers by applying properties of operations as strategies to add and subtract rational numbers.
- Use the properties of multiplication, division and fractions to multiply and divide rational numbers.
- Demonstrate the distributive property with signed numbers and apply to real world situations.
- Apply division to integers with divisors other than zero, recognizing the quotient of integers is a rational number, and apply to real life situations.
- Using prior knowledge of properties of operations, multiply and divide rational numbers.
- Use long division to convert a rational number to a decimal, recognizing the decimal form of rational numbers ends in 0 or repeats.
- Use the four operations with rational numbers to solve real-world and mathematical problems, including the ability to manipulate complex fractions.
- Demonstrate understanding that every number has a decimal expansion of some kind and that rational numbers end in 0's or eventually repeat.
- Compare the size and value of irrational numbers by using rational number approximations, locate them on a number line diagram, and provide an approximate estimation of value.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes (i.e., given figures for a family budget, determine which of a list of bills can be paid without creating a deficit and identify deficit amount if all were paid)
- Text: Steck-Vaughn and Contemporary's Unit Reviews and/or exercises as quizzes and chapter tests
- Chapter/unit reviews/tests from core texts such as New Readers Press *Math Sense*. (See below)

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Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students work in groups of four to reduce a recipe and record new measurements
- Write a real-life math problem involving hourly wages with weekly pay
- Have students use a four-function calculator to check answers
- Use integers to keep track of scores in an interactive football game
- Analyze temperatures over the last four years. Students work in groups choosing different areas from any continent
- Have students rewrite recipes to increase number of servings
- Use interactive technology to practice additive inverse operations
- Have each student make up Scientific Cards in two different colors, one color for standard decimal notation and the other for scientific notation
- Use cards for memory game or matching game
- Have students make a number line using irrational numbers putting them in sequential order

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology: Study Stack, websites as assigned
- One-to-one intervention
- Peer tutoring
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Think out loud (demonstrate how to think about a problem)
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials

- Large number line
- Chart of unit rates, such as ounces, pounds, gallons, hours, minutes, etc.
- Geoboards
- Colored index cards
- Calculators

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- Math fraction manipulatives
- Scientific Notation math chart
- *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
- *Possible Texts*
 - *Math 1: Whole Numbers, Decimals, Fractions, Percents and Measurement.* (2014) New Readers Press/Pre-High School Equivalency
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- Cool Math— www.coolmath.com
- Khan Academy— <http://khanacademy.org/>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

EXPRESSIONS AND EQUATIONS (EE)

4.EE.1 / 4.EE.2 / 4.EE.3 / 4.EE.4 / 4.EE.5 / 4.EE.6 / 4.EE.7 / 4.EE.8 / 4.EE.9 /
4.EE.10 / 4.EE.11 / 4.EE.12

Essential Understandings:

- Variables within algebraic expressions are a modeling tool to use when solving real-world problems. This process demonstrates a method of describing quantitative relationships – for instance, traveling some distance (d) at a given rate of travel will take a given amount of time (t) with a constant rate.
- Solving equations is a reasoning process and follows established procedures based on properties.
- The value of any real number can be represented in relation to other real numbers such as with decimals converted to fractions, scientific notation and numbers written with exponents (e.g., $8 = 2^3$.)
- Understand the difference between an expression and an equation: expressions are simplified and equations are solved for the variable's value.
- Properties of operations can be used to add, subtract, factor, and expand linear expressions.
- Expressions can be manipulated to suit a particular purpose to solve problems efficiently.
- Mathematical expressions, equations, inequalities and graphs are used to represent and solve real-world and mathematical problems.

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- Properties, order of operations, and inverse operations are used to simplify expressions and solve equations efficiently.
- Unit rates can be explained in graphical representations and algebraic equations.
- The solution to a system of two linear equations in two variables is an ordered pair that satisfies both equations.
- Some equations/inequalities and systems of equations/inequalities have no solutions (parallel lines) and others have infinite solutions (same line).
- Square roots and cube roots of small perfect squares and cubes can be evaluated and/or represent solutions to the equations in the form of $x^2 = y$ and $x^3 = y$ where y is a positive rational number.
- The properties of integer exponents can generate equivalent numerical expressions.

Essential Questions:

- What is equivalence? (in reference to expressions and equations)
- How are properties of operations used to prove equivalence?
- When and how are expressions, equations, inequalities and graphs applied to real world situations?
- How can the order of operations be applied to evaluating expressions, and solving from one-step to multi-step equations?
- What are some possible real-life situations to which there may be more than one solution?
- How does the ongoing use of fractions and decimals apply to real-life situations?
- What makes a solution strategy both efficient and effective?
- How is it determined if multiple solutions to an equation are valid?
- How does the context of the problem affect the reasonableness of a solution?
- Why can two equations be added together to get another true equation?
- How can the equations in the form of $x^2 = y$ and $x^3 = y$ where y is a positive rational number be evaluated?
- What is the significance of scientific notation for very large or very small numbers within problem solving situations?

Student will be able to:

- Add, subtract, factor, and expand linear expressions with rational coefficients by applying properties of operations as strategies.
- Write, read, and evaluate expressions involving variables by showing that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. (4EE.11)

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- Demonstrate understanding of expressions by rewriting an expression in a new form to clarify the problem and the way in which quantities are related to each other.
- Use tools strategically to solve multi-step real-life and mathematical problems with positive and negative rational numbers in any form, apply properties of operations as strategies to calculate, converting between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation.
- Using variables to represent real-world or mathematical quantities, construct simple equations and inequalities to solve problems by reasoning about quantities.
- Construct simple equations and inequalities to solve problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are rational numbers, and then compare an algebraic solution to an arithmetic solution and identify the sequence of operations used.
- Construct simple equations and inequalities to solve problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are rational numbers, and graph and interpret the context of the problem.
- Generate equivalent numerical expressions by understanding and applying properties of integer exponents.
- For equations such as $x^2 = p$ and $x^3 = p$, represent solutions by using square root and cube root symbols, evaluating square roots of small perfect squares and cube roots of small perfect cubes.
- Estimate very large or very small quantities and express how many times as much one is than another by using numbers expressed in the form of single digit times an integer power of 10.
- Solve problems with numbers using scientific notation and choose units of appropriate size for measurements of quantities both very large and very small.
- Demonstrate understanding of proportional relationships and lines by graphing proportional relationships, classifying the slope of a line in unit rate, and comparing two proportional relationships represented in different ways.
- Demonstrate understanding of proportional relationships and linear equations by using similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line in the coordinate plane.
- Derive $y = mx$ for a line passing through the origin, and derive $y = mx + b$ for a line intercepting the y -axis at b .
- Demonstrate understanding of single variable equations by providing examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions, and show which possibility is true by converting the given equation to a simpler form.

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- Analyze and solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions with the distributive property and combining like terms.
- Evaluate and solve pairs of simultaneous linear equations by understanding that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- Evaluate and solve pairs of simultaneous linear equations by solving systems of two linear equations in two variables algebraically, and estimate by graphing the equations.
- Evaluate and solve pairs of simultaneous linear equations by solving real-world and mathematical problems leading to two linear equations in two variables.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes such as use substitution to determine monthly car payments based on a four-year plan (See class activities)
- Text: Unit Reviews and/or exercises as quizzes and chapter tests
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Estimate weekly wages based on commission
- Use substitution to determine monthly car payments based on a three-year plan
- Study price of gas, calculating percentage of increase/decrease
- Write a multi-step equation based on a present hourly wage with a 10% raise. Calculate the new hourly pay
- Write an inequality based on how many hours a person need to work per week if they earn x per hour with y axis

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- Using information from a real-world problem, graph prop
- Proportional relationships, interpreting the unit rate as the slope of a graph
- Estimate the population of endangered species, writing it in scientific notation
- Study price of gas, calculating percentage of increase/decrease
- Write a multi-step equation based on a present hourly wage with a 10% raise. Calculate the new hourly pay
- Write an inequality based on how many hours a person needs to make a week to satisfy a given budget
- Using information from a real-world problem, graph proportional relationships, interpreting the unit rate as the slope of a graph
- Estimate the population of endangered species, writing it in scientific notation

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology: Study Stack, websites as assigned
- One-to-one intervention
- Peer tutoring
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Think out loud (demonstrate how to think about a problem)
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Spread sheets
- Calculators
- Multiplication boxes
- Counters
- Fictional budgets and financial records
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *EMPower Math, Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Math Sense 2: Algebraic Thinking, Data Analysis, and Probability.* (2014). New Readers Press.
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

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List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Kuta Software— <https://www.kutasoftware.com>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

FUNCTIONS (F)

4.F.1 / 4.F.2 / 4.F.3 / 4.F.4 / 4.F.5

Essential Understandings:

- A function is a specific topic of relationship in which each input has a unique output that can be represented in a table.
- A function can be represented graphically using ordered pairs that consist of the input and the output of the function in the form (input, output).
- A function can be represented with an algebraic rule.
- The equation $y = mx + b$ is a straight line and that slope and y-intercept are critical to solving real problems involving linear relationships.
- Changes in varying quantities are often related by patterns that can be used to predict outcomes and solve problems.
- Linear functions may be used to represent and generalize real situations.

Essential Questions:

- How do ordered pairs on coordinate graphs help define relationships?
- What defines a function and how can it be represented?
- What makes a function linear?
- How can linear relationships be modeled and used in real-life situations?
- Why is one variable dependent upon the other(s) in relationships?

Student will be able to:

- Define a function by explaining that the graph of a function is the set of ordered pairs consisting of an input and a corresponding output.
- Compare functions by considering properties of two functions that are represented algebraically, graphically, numerically, or verbally.
- Evaluate functions by demonstrating that $y = mx + b$ represents a linear equation as opposed to functions that are non-linear.
- Model relationships between two quantities by creating a function, determine the rate of change and value of the function from a description of given (x,y) values, and interpret and find the rate of change and initial value of a function by reading from a graph.
- Model relationships between two quantities by analyzing a graph where the function is increasing or decreasing, linear or nonlinear.
- Model relationships between two quantities by sketching a graph showing the qualitative features of a function based on a verbal description.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes: interpret two growth charts
- Textbook Unit Reviews and/or exercises as quizzes and chapter tests
- Chapter/unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Compare graphs on social media usage in the last ten years
- Read growth charts, comparing American child growth to that in third world countries
- Interpret nutritional graphs indicating data from fast food restaurants

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Rulers
- Graphing Calculators- Desmos and TI30Xs
- Posters containing real world graphed information, (i.e., Nutrition, financial)
- Newspaper graphs

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- *Possible Text:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *EMPower Math, Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). McGraw-Hill.
 - *Math Sense 2: Algebraic Thinking, Data Analysis and Probability* (2014). New Readers press
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Kuta Software— <https://www.kutasoftware.com>
- Vocabulary support: www.studystack.com/
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

GEOMETRY (G)

4.G.1 / 4.G.2 / 4.G.3 / 4.G.4 / 4.G.5 / 4.G.6 / 4.G.7 / 4.G.8 / 4.G.9 / 4.G.10 /
4.G.11 / 4.G.12 / 4.G.13 / 4.G.14

Essential Understandings:

- Scale drawings can be applied to problem solving situations involving geometric figures.
- Geometrical figures can be used to reproduce a drawing at a different scale
- Algebraic reasoning is applied when solving geometric problems (i.e., circumference and area of a circle).
- Unit rates can be explained in graphical representation, algebraic equations, and in geometry through similar triangles.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real-life experiences.
- Properties of two-dimensional shapes are used in solving problems involving three-dimensional shapes.
- Planes that cut poly-hedra create related two-dimensional figures. Reflections, translations, and rotations are actions that produce congruent geometric objects.
- Dilations, translations, rotations and reflections can be shown using two-dimensional figures on a coordinate plane.
- A dilation is a transformation that changes the size of a figure but not the shape.

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- Two similar figures are related by a scale factor, which is the ratio of the lengths of corresponding sides.
- If the scale factor of a dilation is greater than 1, the image resulting from the dilation is an enlargement, and if the scale factor is less than 1, the image is a reduction; both transformations result in similar figures.
- A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of transformations.
- Two shapes are similar if the length of all the corresponding sides are proportional and all the corresponding angles are congruent.
- Congruent figures have the same size and shape (a rigid, fixed relationship). If the scale factor of dilation is equal to 1, the image resulting from the transformation is a congruent figure.
- Right triangles have a special relationship among the side lengths that can be represented by a model and a formula.
- The Pythagorean Theorem and its converse can be proven.

Essential Questions:

- Why is it important to use precise language and mathematical tools in the study of two-dimensional and three-dimensional figures?
- How do graphs illustrate proportional relationships?
- How are geometric figures used to reproduce a drawing at a different scale?
- What models on the coordinate plane are helpful for understanding and quantifying the volume of rectangular prisms?
- How does investigating figures help us build our understanding of mathematics?
- How can proportional relationships of congruent and similar figures be used to solve ratio problems?
- How are scale drawings used to compute actual lengths and area?
- What are transformations and what effect do they have on an object?
- What does the scale factor of a dilation convey?
- How can transformations be used to determine congruency or similarity?
- What angle relationships are formed by a transversal intersecting with two parallel lines?
- Why does the Pythagorean Theorem apply only to right triangles?
- How does the knowledge of how to use right triangles and the Pythagorean Theorem enable the design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads?
- How do indirect measurement strategies (using the Pythagorean Theorem) allow for the measurement of items in the real world such as playground structures, flagpoles, and buildings?
- How is the volume and/or surface area of various three-dimensional geometric objects determined?

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Middle Intermediate Basic Education (Grade Levels 6-7)

Student will be able to:

- Draw, construct and describe geometrical figures by solving problems with scale drawings and computing actual lengths and areas from a scale drawing as well as reproducing a scale drawing at a different scale.
- Draw, construct and describe geometric shapes with given conditions by focusing on triangle construction from three angle or side measures and recognizing what conditions determine particular triangles, more than one triangle, or no triangle.
- Construct and describe geometrical figures by slicing three-dimensional figures, such as plane sections of right rectangular prisms or right rectangular pyramids.
- Solve real-world and mathematical problems involving measurements of angles, area, surface area, and volume through application of formulas for area and circumference of a circle as well as identifying the relationship between a circle's area and its circumference.
- Solve real-life and mathematical problems pertaining to angle measure by applying rules of supplementary, complementary, vertical and adjacent to multi-step problems with equations for unknown angle in a figure.
- Solve real-life and mathematical problems pertaining to area, surface area, and volume for two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- Demonstrate knowledge of congruence and similarity by verifying experimentally the properties of rotations, reflections, and translations through identifying that lines are taken to lines and line segments to line segments of the same length.
- Demonstrate knowledge of congruence and similarity by verifying experimentally the properties of rotations, reflections, and translations through observing that angles are taken to angles of the same measure.
- Demonstrate knowledge of congruence and similarity by verifying experimentally the properties of rotations, reflections, and translations by observing that parallel lines are taken to parallel lines.
- Demonstrate knowledge of congruence by showing that a pair of two-dimensional figures are congruent if the second can be obtained from the first, and describe a sequence that exhibits the congruence between the two figures.
- Use coordinates to demonstrate the effects of dilations, translations, rotations, and reflections on plane figures.
- Demonstrate knowledge of similarity by showing that a pair of two-dimensional figures are similar if the second can be obtained from the first, and describe a sequence that exhibits the similarity between the two figures.
- Develop facts about angle sums and exterior angles of triangles, angles created by parallel lines cut by transversals, and the angle-angle criterion.
- Identify and explain a proof of the Pythagorean Theorem and its converse.

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Middle Intermediate Basic Education (Grade Levels 6-7)

- Solve real-world and mathematical problems for unknown side lengths in two- and three-dimensional geometric shapes containing right triangles by applying the Pythagorean Theorem.
- Find the distance between two points in a coordinate system by applying the Pythagorean Theorem.
- Demonstrate knowledge of formulas for volume of cylinders, cones, and spheres by applying them to and finding solutions for real-world and mathematical problems.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes: Sketch geometric shapes as directed and divide in half
- Text book Unit reviews or tests and/or exercises as quizzes and chapter tests
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work
- Exit slips, math minutes, etc.

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Estimate the amount of tile needed to cover a 12ft² floor.
- Have students plan a garden in different shape and size planters, finding the volume of soil needed to fill each pot. Chart findings to compare results
- Sketch geometric shapes, extend activity by slicing them in half or thirds
- Have students create three-dimensional solid rectangular objects using Paper Mache. Then have students use their rectangular solids to calculate its total surface area *or from templates available at <https://www.math-salamanders.com/3d-geometric-shapes.html>*
- Have students make picture frames of various shapes, cutting them from stock paper. This will involve precise angle measurement and cutting to fit edges together

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Using hand sketched rotations, reflections, dilations and translations, write the effects on two-dimensional figures using coordinates
- Tie a rope to a basketball net and stretch it out to form an angle. Have students apply the Pythagorean Theorem to find the distance from the net to the ground
- Challenge students to bring in examples of cylinders, cone, and spheres. Discuss the formulas for each volume and apply them to each item brought in by students

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Advertisements for prices on tile, carpet, (based on sq. ft.), and bags of soil
- Interactive technology math sites
- Stock paper
- Protractors
- Rulers
- Scissors
- Rope for creating angles
- Calculators
- Solid three-dimensional objects that can be used to find surface area
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *EMPower Math, Over, Around, and Within: Geometry and Measurement, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Geometry and Measurement* (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

List of Technology Resources:

- GeoGebra <https://www.geogebra.org/?lang=en>
- Khan Academy— <http://khanacademy.org/>
- Study Stack --- www.studystack.com
- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

STATISTICS AND PROBABILITY (SP)

4.SP.1 / 4.SP.2 / 4.SP.3 / 4.SP.4 / 4.SP.5 / 4.SP.6 / 4.SP.7 / 4.SP.8 / 4.SP.9

Essential Understandings:

- Bivariate categorical data display frequencies and relative frequencies can be seen in two-way tables.
- Random sampling tends to produce representative samples and support valid inferences.
- The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
- The probability of a chance event is approximated by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the approximate relative frequency given the probability.
- A probability model, which may or may not be uniform, is used to find probabilities of events.
- Various tools are used to find probabilities of compound events (including organized lists, tables, tree diagrams, and simulations).
- Written descriptions, tables, graphs, and equations are useful in representing and investigating relationships between varying quantities.
- Slope and y-intercept are keys to solving real problems involving linear relationship models of data.
- Some data may be misleading based on representation.

Essential Questions:

- Using measures of central tendency, how are data sets interpreted and analyzed?
- How are probability and the likelihood of an occurrence related and represented?
- How is probability approximated?
- How is a probability model used?

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Middle Intermediate Basic Education (Grade Levels 6-7)

- How are probabilities of compound events determined?
- What relationships can be seen in bivariate categorical data?
- What conclusions can be drawn from data displayed on a graph?
- What do the slope and y-intercept of a line of best fit signify on a graph? What do outliers signify?
- How can graphs, tables, or equations be used to describe patterns and predict subsequent data or outcomes?

Student will be able to:

- Summarize numerical data sets in relation to their context by reporting the number of observations.
- Summarize numerical data sets in relation to their context by describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- Summarize numerical data sets in relation to their context by giving quantitative measures of center and variability, as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data was gathered.
- Summarize numerical data sets in relation to their context by relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data was gathered.
- Use random sampling to draw inferences about a population with an unknown characteristic, and generate multiple samples of the same size to gauge the variation in estimates or predictions.
- Use random sampling to draw inferences about a population by using measures of center and measures of variability to draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models by demonstrating that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
- Investigate chance processes and develop, use, and evaluate probability models by approximating the probability of a chance event by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the relative frequency given the probability.
- Investigate chance processes and develop, use, and evaluate probability models by comparing probabilities from a model to observed frequencies and if the agreement is not good, explain possible sources of the discrepancy.
- Develop a probability model by assigning equal probability to all outcomes and using the model to determine probabilities of events, as well as by observing frequencies in data generated from a chance process.
- Use organized lists, tables, tree diagrams, and simulation to find probabilities of compound events, demonstrating why the probability of a

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Middle Intermediate Basic Education (Grade Levels 6-7)

compound event is the fraction of outcomes in the sample space for which the compound event occurs.

- Use organized lists, tables, and tree diagrams to find probabilities of compound events, representing sample spaces for compound events, and design a simulation generating frequencies for compound events.
- Investigate patterns of association in bivariate data by constructing and interpreting scatter plots for bivariate measurement data in order to investigate patterns of association between two quantities, classifying patterns such as clustering, outliers, positive or negative association, linear or nonlinear association.
- Investigate patterns of association in bivariate data by demonstrating why straight lines are widely used to model relationships between two quantitative variables.
- Investigate patterns of association in bivariate data by using equations of linear models to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- Investigate patterns of association in bivariate data by constructing and interpreting a two-way table summarizing data on two categorical variables collected from the same subjects, using relative frequencies for rows or columns to describe possible association between the two variables.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher made assignments and quizzes (See class activities)
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students work in groups to collect data on a topic of each group's choice. Each group should display numerical data in a different way, for example: one group could plot on a number line, another group on dot plots, and still another using box and whisker

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- Adding to this classroom activity, students must explain in their math journals the distribution on the data collected
- Collect data from students to determine associations between test scores and sleep, coffee and sleep, stress and sleep, etc.
- Plot and analyze related student attributes (i.e., bed times and wake up times, workdays per week and times eating out per week, number of children, number of hours of sleep per night)
- Survey two different populations from opposite climates and take random samples to draw informal comparative inferences. Each student can measure the center and variability for the numerical data collected
- Play a probability game using dice or playing cards. Students can roll the dice several times and keep track of the chances it produces and its frequency over time
- Students can do the same with tossing a coin. Students are to record all findings in their math journal
- A third activity will be using coins and dice for compound events.
- All activities should be recorded on tables, tree diagrams, and on organized lists
- As a whole group, plot and analyze each student's daily routines and use this information to understand and explain patterns of association in bivariate data
- Then use a linear equation to solve problems in the context of a bivariate measurement and interpret the slope and intercept
- Students then record frequencies and relative frequencies in a two-way table

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One on one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Newspaper (local and world)
- Chart paper, graph paper

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- Interactive technology
- Dice (6-sided, 10-sided, etc.), coins, cards, rulers
- Markers and colored pencils
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *Math Sense 2: Algebraic Thinking, Data Analysis, and Probability* (2014). New Readers Press
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 5**

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NRS Level 5
Low Adult Secondary Education (Grade Levels 7 – 8)

NUMBER AND QUANTITY (N)
THE REAL NUMBER SYSTEM (RN)

5.N.RN.1 / 5.N.RN.2 / 5.N.RN.3 / 5.N.RN.4 / 5.N.RN.5 / 5.N.RN.6

Essential Understandings:

- Adding and multiplying two rational numbers results in a rational number.
- The result of adding a rational number and an irrational number is an irrational number.
- The result of multiplying a non-zero rational number to an irrational number is an irrational number.

Essential Questions:

- What is the relationship between properties of operations and types of numbers?
- Why are quantities represented in multiple ways?
- How can quantities be represented and what is the rationale for selecting a specific representation?
- What type of number results when adding or multiplying two rational numbers?
- What type of number results when adding a rational number to an irrational number?
- What type of number results when multiplying a non-zero rational number to an irrational number?

Student will be able to:

- Use prior knowledge of operations to add and subtract rational numbers and graph results on a line diagram.
- Apply and extend previous understanding of addition and subtraction of rational numbers by describing situations in which opposite quantities combine to make 0.
- Apply and extend previous understanding of addition and subtraction of rational numbers by showing understanding of $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative.
- Demonstrate that a number and its opposite are additive inverses, so their sum is 0, and interpret sums of rational numbers in real-world contexts.
- Apply and extend previous understanding of addition and subtraction of rational numbers by demonstrating the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- Demonstrate previous knowledge of addition and subtraction of rational numbers by applying properties of operations as strategies to add and subtract rational numbers.
- Use the properties of multiplication, division and fractions to multiply and divide rational numbers.
- Demonstrate the distributive property with signed numbers and apply to real world situations.

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- Apply division to integers with divisors other than zero, recognizing the quotient of integers is a rational number, and apply to real life situations.
- Use long division to convert a rational number to a decimal, recognizing the decimal form of rational numbers ends in 0 or repeats.
- Demonstrate understanding that every number has a decimal expansion of some kind and that rational numbers end in 0's or eventually repeat.
- Compare the size and value of irrational numbers by using rational number approximations, locate them on a number line diagram, and provide an approximate estimation of value.
- State or write (using words or examples) the difference between a rational and irrational number
- Add and multiply two rational numbers to obtain a rational number.
- Add a rational and irrational number to obtain an irrational number.
- Multiply a nonzero rational number and an irrational number to obtain an irrational number.
- Use the four operations with rational numbers to solve real-world and mathematical problems, including the ability to manipulate complex fractions.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes
- Textbook unit reviews and/or exercises as quizzes and chapter tests

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Extend examples of relationships between exponents and radicals through interactive technology
- Use practice worksheets to develop skills of rational exponent usage
- Students will work in groups, reasoning about general statements on rational and irrational numbers
- Students record thinking in their math journals: explaining and justifying their classification decisions
- Students should work independently to improve their individual solutions to the assessment task

NRS Level 5 Low Adult Secondary Education (Grade Levels 7 – 8)

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

(Core and supplemental)

- Mini white boards
- Calculators
- Possible Texts:
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *Common Core Achieve: Mathematics.* (2013). Contemporary/McGraw-Hill

List of Technology Resources:

- Kuta Software—<https://www.kutasoftware.com>
- Purple Math—www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

NUMBER AND QUANTITY (N)

QUANTITIES (Q)

5.N.Q.1 / 5.N.Q.2

Essential Understandings

- Rates, ratios, percentages and proportional relationships can be applied to multi-step ratio and percent problems along with other problem solving situations such as interest, tax, discount, etc.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Arithmetic and algebra can be used together, with the rules of conversion to transform units.
- Scales, graphs, and other data models can be interpreted.

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Essential Questions:

- When is it advantageous to represent relationships between quantities symbolically? numerically? graphically?
- How can the units used in a problem help determine a solution strategy?
- How can units, scale, data displays and levels of accuracy be chosen to appropriately represent a situation?

Student will be able to:

- Express the relationships between quantities symbolically, numerically, graphically and verbally when given a real-world situation or a mathematical context.
- Use appropriate units when obtaining an arithmetic or algebra solution to a real-world multi-step problem.
- Use and interpret appropriate units consistent with a given formula or multi-step problem (e.g., area will have square units and volume with cubed units).
- Choose and interpret scales and the origin on various types of graphs and/or data displays.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from texts
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities. (See class activities for possibilities)
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Make and interpret graphs, such as circle graph, bar graph, and line graph, based on real-life data (candy, student groups represented, classroom objects)
- Students will make frequency tables to go with their graphs
- Have students apply multiple discounts to a given item
- Have students calculate the rate of change in real-life situations. For example, calculate the change in attendance at the state fair from one year to another.

Learning Activities: (*differentiated instruction, re-teaching options*)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention

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- Peer teaching through group work
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Small packet of candies or lollipops
- Colored pencils or markers
- Poster board
- Rulers and glue
- Store advertisements
- *Mathematical Reasoning: Test Preparation for the 2014 GED TEST.* (2014). Steck- Vaughn.
- *Possible Texts:*
 - *Math 2: Algebraic Thinking, Data Analysis, and Probability* (2014). New Readers Press
 - *Data Analysis and Probability* (2016). McGraw Hill/College and Career Readiness) Practice Workbook.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways

ALGEBRA (A) SEEING STRUCTURE IN EXPRESSIONS (SSE) 5.A.SSE.1

Essential Understandings:

- Identify and interpret the different parts of expressions that represent certain values contextually.
- Exponential expressions represent a quantity in terms of its context.

Essential Questions:

- What are the different ways to represent an exponential expression?
- How can an appropriate expression be created to model data or situations given within context?

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Student will be able to:

(what does mastery look like)

- Identify and interpret a term, its factors and its coefficient within any polynomial expression.
- Interpret expressions having grouping symbols by viewing one of more of the factors or parts as a single entity.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities (See class activities for possibilities)
- Assign outside projects as evaluated against a rubric
- Math journals-graded entries
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Students will work in groups to create mock financial records. Each member of the group will contribute ideas for keeping personal financial records
- Students will use the concept of exponential growth to model functions in real-life situations (i.e., the spread of colds and bacteria)

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Focus on a clear statement of an “exponential relationship”
- Have students graph points on a smart board or other interactive technology with teacher direction
- Have students make comparisons between linear and exponential graphs
- Have them write all findings in their math journal
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts

NRS Level 5 Low Adult Secondary Education (Grade Levels 7 – 8)

- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers
- Legal pads, graphing paper
- Play money
- Markers
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw Hill/College and Career Readiness Practice Workbook
 - *EMPower Math: Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

ALGEBRA (A) CREATING EQUATIONS (CED) 5.A.CED.1 / 5.A.CED.2 / 5.A.CED.3 / 5.A.CED.4

Essential Understandings:

- Linear models can be created, used, and interpreted for real-life situations.
- Real world situations can be modeled by systems of linear equations.
- A system of equations can have no, one, or infinitely many solutions.
- Solutions of systems of inequalities are ordered pairs that satisfy all equations as well as inequalities that are often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Multiple methods may be used to solve a system of equation or inequalities.
- Functions can be created to best fit data represented on various models.
- Polynomial functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation.
- Systems can be solved graphically, algebraically or from a table.

Essential Questions:

- What real world situations can be modeled by a linear relationship?

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- How can technology help to determine whether a linear model is appropriate in a given situation?
- How can systems of linear equations or inequalities be used to model real world situations?
- How can the solution(s) of a system be represented and interpreted?
- What processes may be used to solve a system of equations or inequalities?
- How can a linear function be found that best fits data from various models?
- What are the different methods that can be used to find the solutions of a system of equations?
- When changes are made to an equation, what changes are made to the graph?
- What new information will be revealed if a formula is written in a different but equivalent form?
- How can the solution(s) of a system be represented and interpreted?

Student will be able to:

(what does mastery look like)

- Create and interpret various types of equations and inequalities in one variable, using them to solve problems.
- Create and interpret various types of equations in two or more variables, describing the relationship of the two quantities being represented and determining whether the solution(s) are viable or nonviable for the modeling context.
- Graph systems of either equations or inequalities on a coordinate axes, properly labeling and scaling the axes.
- Graph systems of equations or inequalities on a coordinate axes, properly labeling and scaling the axes.
- Rewrite formulas or literal equations to highlight a quantity of interest.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

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- Students survey different word problems that involve writing an equation with one variable. Then they write their own word problem that can be solved by a two-step equation.
- Students then experiment with different methods of solving systems of equations
- Demonstrate graphing equations and inequalities on a coordinate plane.
- Students experiment with graphing sample equations and inequalities in small groups

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring through group work
- Think out loud (demonstrate how to think about a problem)
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Straight edge
- Pencils
- Formula worksheet from GED® testing service:
<http://www.gedtestingservice.com/uploads/files/0756c16704434ff71e43c8117a5fa738.pdf>
- Possible Texts:
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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ALGEBRA (A)
REASONING WITH EQUATIONS AND INEQUALITIES (REI)
5.A.REI.1 / 5.A.REI.2 / 5.A.REI.3 / 5.A.REI.4 / 5.A.REI.5

Essential Understandings:

- Algebraic concepts can be proven, and actions taken to arrive at a solution can be justified.
- The relationships between quantities can be explained or justified verbally in the exploration of real-world situations.
- The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous).
- Linear functions can be represented by a table, graph, verbal description or equation and that each representation can be transferred to another representation.
- Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).
- There are several ways to solve a quadratic equation (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the equation.
- The quadratic formula is derived from the process of completing the square.
- Complex numbers exist and can arise in the solutions of quadratic equations.
- A quadratic function that does not intersect the x -axis has complex zeros.
- The relationship between the factors of a quadratic and the x -intercepts of the graph of the quadratic.

Essential Questions:

- Why are procedures and properties necessary when manipulating numeric or algebraic expressions?
- How can the structure of an equation or an inequality help determine a solution strategy?
- What are complex numbers, and why do they exist?
- How can a quadratic equation be solved?
- How do the factors of a quadratic determine the x -intercepts of the graph and vice versa?
- How is the quadratic formula derived?

Student will be able to:

(what does mastery look like)

- Write a viable argument to justify each step used to find a solution in a simple equation.
- Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.
- Use the method of completing the square to derive the quadratic formula.

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- Solve quadratic equations in one variable for its zeros by either factoring, completing the square, using the quadratic formula, or using the square root property, recognizing that a solution(s) can be either real, irrational, or complex (written in a $\pm bi$ form).
- Use either the method of substitution or elimination to solve systems of equations.
- Find either an exact or approximate solution, focusing on systems of linear equations.
- Represent and solve equations and inequalities in two variables in the coordinate plane, explaining the set of all the solutions plotted.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Students work in groups of four. Each group has a bell. An equation is put on the Smart Board or other interactive technology. (This activity can be used for one-step, multiple steps, equations with two variables, or inequalities.) Groups ring the bell when they know the first step to solving the equation. The group to ring first and has the right answer earns a point for their team. They can converse with their group but have 30 seconds to answer. This continues until the problem is solved
- Have students answer concept questions in their math journals that involve systems of two linear equations in two variables. For example: 1) What can you say about the solution? 2) Explain what it means for a system to be independent. 3) Explain what it means for the system to be inconsistent
- Have students participate in self-check exercises working with a partner, followed by a discussion of all steps toward solutions

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on articulating a clear statement on “equation relationships”
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as:

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interactive websites/illustrations, Study Stack – vocabulary support

- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Rulers
- Pencils
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *EMPower Math: Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

FUNCTIONS (F) INTERPRETING FUNCTIONS (IF) 5.F.IF.1

Essential Understandings:

- The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous).
- The zeros of each factor of a polynomial determine the x -intercepts of its graph.
- Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).

Essential Questions:

- How can a function and its notation be used, interpreted, and defined?
- How can you represent a function and what are the key features of each representation?
- What are the key features of a linear or quadratic function? Slope? Intercepts? Maxima? Minima?
- What type of linear, quadratic or exponential function is best to model a given situation?

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Student will be able to:

(what does mastery look like)

- Use paper-and-pencil to graph simple functions and use technology to graph more complicated functions, showing key features of the graph.
- Graph linear functions showing intercepts.
- Graph quadratic functions showing intercepts and either maxima or minima.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on graphing quadratic functions, focusing on the skill of creating tables and graphs.
- Introduce quadratics and the general form of the quadratic function and equation, demonstrating how to convert the quadratic equation into the quadratic function. Use a graph to give students a visual representation of what the key terms represent.
- Next show students how to graph two simple quadratic functions (i.e., $y=x^2$ and $y=3x^2$)
- Add additional examples in increasing difficulty and with negative coefficients for the quadratic terms.
- Present students with opportunities to participate in the lesson by encouraging them to come to the board to fill out the table of values or graph the quadratic function.

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on articulating a clear statement on “equation relationships”
- Teacher- directed class discussion on key features of the quadratic graph.
- Have students make comparisons between quadratic graphs and linear graphs
- Teaching through group work

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- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers and graph paper
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - *EMPower Math: Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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FUNCTIONS (F)
BUILDING FUNCTIONS (BF)
5.F.BF.1 / 5.F.BF.2

Essential Understandings:

- Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line.
- Arithmetic sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.

Essential Questions:

- What is an arithmetic sequence and how does it relate to linear functions?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- How can applied problems using quadratics be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (e.g., factoring to find the zeros, or completing the square to find the maximum or minimum)?

Student will be able to:

- Write a function and describe the relationship between the two quantities represented.
- Find an explicit expression or a recursive process and describe the steps for calculating an expression from a context.
- Create a linear or exponential function or an arithmetic or geometric sequence, given a graph, a verbal description or an input-output table, transferring easily between each of these representations to obtain a correct solution.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Math journals-graded entries
- Facilitated group work/ monitoring conversations and work with rubric

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present lesson on functions with a skill focus on creating tables, graphs, and equations
- Review key vocabulary: relationship, function, independent variable, dependent variables, linear relationship, linear graph, x-axis, y-axis, ordered pairs, explicit expression, recursive process
- On the overhead, show different types of functions that clearly describe a relationship between quantities
- Ask the questions: Where in real life can situations be found where the value of one variable depends on the value of another? How does changing the value of the independent variables affect the value of the dependent variables?
- Explain that the recursive process uses the previous term and the rate of change and that an explicit expression uses the rate of change and the initial value

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on articulating a clear statement on “equation relationships”
- Teacher-directed class discussion on key features of the quadratic graph.
- Have students make comparisons between quadratic graphs and linear graphs
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One on one intervention
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Transparency with different types of graphs
- Calculator
- Graph paper
- Rulers
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016) McGraw-Hill/College and Career Readiness Practice Workbook
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

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List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) CONGRUENCE (CO)

5.G.CO.1 / 5.G.CO.2 / 5.G.CO.3 / 5.G.CO.4 / 5.G.CO.5 / 5.G.CO.6 / 5.G.CO.7 /
5.G.CO.8 / 5.G.CO.9 / 5.G.CO.10 / 5.G.CO.11 / 5.G.CO.12 / 5.G.CO.13

Essential Understandings:

- When parallel lines are cut by a transversal, corresponding angles, alternate interior angles, alternate exterior angles, and vertical angles are congruent.
- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects represented in the coordinate plane.
- Any two geometric figures are congruent if there is a sequence of rigid motions (rotations, reflections, or translations) that carries one onto the other.
- A proof consists of a hypothesis and conclusion connected with a series of logical steps.
- The basic building blocks of geometric objects are formed from the undefined notions of point, line, distance along a line, and distance around a circular arc.
- Two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles of the triangles are congruent.
- It is possible to prove two triangles congruent without proving corresponding pairs of sides and corresponding pairs of angles of the triangle are congruent if certain subsets of these six congruence relationships are known to be true (e.g. SSS, SAS, ASA, but not SSA).
- Different observed relationships between lines, between angles, between triangles, and between parallelograms are provable using basic geometric building blocks and previously proven relationships between these building blocks and between other geometric objects.
- The geometric relationships that come from proving triangles congruent may be used to prove relationships between geometric objects.
- Geometric figures can be constructed using various tools, methods and relationships.

Essential Questions:

- In terms of rigid motions, when are two geometric figures congruent?
- What are the undefined building blocks of geometry and how are they used?
- What are possible conditions that are necessary to prove two triangles congruent?
- What are the roles of hypothesis and conclusion in a proof?
- What criteria are necessary in proving a theorem?

NRS Level 5

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- What is the significance of demonstrating the relationships between geometric figures through constructions?

Student will be able to:

- Write a definition based on the undefined notions of point, line, distance along a line and distance around a circular arc for an angle, a circle, a set of perpendicular or parallel lines, and line segments.
- Develop, describe, and draw a transformation or a sequence of transformations (translations, rotations, and reflections) of a given geometric figure as a function of input and output values or by using the coordinate plane.
- Develop, describe, and draw a transformation or a sequence of transformations (translations, rotations, and reflections) of a rectangle, parallelogram, trapezoid, or regular polygon as it can be carried onto itself by creating an input-output table of values or by using the coordinate plane.
- Compare and contrast a transformation (i.e., a translation versus a horizontal stretch) which preserves and does not preserve the distance and/or angular measure of a geometric figure in the coordinate plane.
- Use geometric descriptions of rigid motions to transform and predict the effect of a given rigid motion onto a given geometric figure.
- Use the definition of congruence in terms of rigid motions to decide if two geometric figures are congruent.
- Explain how the criteria for triangle congruence (i.e., ASA, SAS, SSS) follow the definition of congruence in terms of rigid motion.
- Prove theorems about lines, angles, triangles and parallelograms.
- Use various geometric tools to make formal geometric constructions, such as copying a segment or an angle, bisecting a segment and/or angle, constructing either perpendicular or parallel lines, and state the significance of demonstrating relationships between geometric figures through constructions.
- Construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle with a compass and straightedge.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Give students visuals of congruent triangles accompanied with five postulates and theorems. Use two-column proofs to model all six components that prove congruency. The columns also will model the thought process in a logical and orderly way
- The second model will be a justification of the columns by citing postulates, theorems, or definitions
- Next, present students with a representation of two identical triangles. They must discuss reasons why the triangles are identical (congruent). Students list supporting postulates, theorems, and definitions
- Students work together in small groups, reasoning to draw conclusions and solve problems
- Give student groups clues, fill in blanks, and steps presented in order
- Students complete a worksheet with fifteen sample problems. Each problem is accompanied by a two-column fill-in, with postulates and theorems to choose from

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Teacher-directed class discussion on key features
- Have students make comparisons of different theorems
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Math manipulatives for drawing accurate geometric shapes
- Protractor
- Colored pencils
- Overhead projector
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Geometry and Measurement* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

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List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G)

SIMILARITY, RIGHT TRIANGLES AND TRIGONOMETRY (SRT)

5.G.SRT.1 / 5.G.SRT.2 / 5.G.SRT.3 / 5.G.SRT.4 / 5.G.SRT.5 / 5.G.SRT.6

Essential Understandings:

-
- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects represented in the coordinate plane.
- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged, and that the dilation of a line segment is longer or shorter in the ratio given by the scale factor of the dilation.
- Two geometric figures are similar if there is a sequence of similarity transformations (dilation along with rotations, reflections, or translations) that carries one onto the other.
- Two triangles are similar if and only if corresponding pairs of angles are congruent and corresponding pairs of sides are proportional.
- It is possible to prove two triangles similar by proving that two pairs of corresponding angles of the triangles are congruent.
- The Pythagorean Theorem can be used to find the missing side lengths in a coordinate plane and real-world situations.
- Different observed relationships between geometric objects are provable using basic geometric building blocks and previously proven relationships between these building blocks and between other geometric objects.
- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects.
- The properties of congruent and of similar triangles can be used to solve problems that either involve or can be modeled with triangles.

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Essential Questions:

- What angle relationships are formed by a transversal intersecting with two parallel lines?
- What are the properties of dilations?
- In terms of similarity transformations, when are two geometric figures similar?
- What are the necessary conditions to know when two triangles are similar?
- What are the sufficient conditions to know that two triangles are similar?
- How can the Pythagorean Theorem be proven using the geometric relationships that come from proving triangles similar?
- How can the geometric relationships that come from proving triangles congruent or from proving triangles similar be applied in problems solving situations?
- How do indirect measurement strategies (using the Pythagorean Theorem) allow for the measurement of items in the real world such as playground structures, flagpoles, and buildings?

Student will be able to:

- Use proper terminology (e.g., reduction and/or enlargement) to describe how the properties of a dilation can be used on a geometric figure given either a center or a scale factor.
- Determine when two geometric figures are similar in terms of similarity transformations.
- Use the properties of similarity transformations to establish the Angle-Angle (AA) criteria for two similar triangles.
- Prove and apply theorems about triangles using congruence and similarity criteria.
- Develop facts about angle sums and exterior angles of triangles, angles created by parallel lines cut by transversals, and the angle-angle criterion.
- Find the distance between two points in a coordinate system by applying the Pythagorean Theorem.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Facilitated group work/ monitoring conversations and work with rubric

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson verifying experimentally the properties of dilations given by a center and scale factor
- Explain that dilations create parallel lines between ALL pre-image and image corresponding segments and lines
- The lesson should move slowly, starting with basic dilations with one ray. It is upon a ray that students will gain understanding of enlargement, reduction, and direction of dilation
- Definition: A dilation with the center O and a scale factor of K is a transformation that maps every point P in the plane P' so that the following properties are true:
 - 1) Angle measure (angles stay the same)
 - 2) Parallelism (parallel lines stay the same)
 - 3) Collinearity (points stay on the same lines)
 - 4) Midpoint (midpoints remain the same)
 - 5) Orientation (lettering order stays the same)
 - 6) Distance (distance is NOT preserved)
- After a dilation, the pre-image and image have the same shape but not the same size, and the dilation leaves a line unchanged.
- Use different worksheets with coordinate planes to give students opportunities to sketch dilation images and practice labeling

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on dilation properties
- Focus on articulating a clear statement on similarity of geometric figures
- Teacher- directed class discussion on key features of Necessary conditions and sufficient conditions of triangle similarity
- Have students make comparisons between the three accepted methods of Proofs with Similar Triangles
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Protractor
- Overhead projector on Smart Board technology for viewing interactive dilations

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- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Geometry and Measurement* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) CIRCLES (C)

5.G.C.1 / 5.G.C.2 / 5.G.C.3 / 5.G.C.4 / 5.G.C.5

Essential Understandings:

- Rounded object volume can be calculated with specific formulas.
- Pi is necessary when calculating volume of rounded objects.
- Different relationships among inscribed angles, radii, and chords of a circle, and between the angles of a quadrilateral inscribed in a circle are provable using previously proven relationships between geometric objects.

Essential Questions:

- What are the different relationships among inscribed angles, radii, and chords of a circle, and of the angles of a quadrilateral inscribed in a circle?
- What is the relationship between the length of the arc of a circle, the central angle of the circle that intercepts the arc, and the radius of the circle?
- What is the area of a sector of a circle?

Student will be able to:

(what does mastery look like)

- Prove that all circles are similar.
- Identify central angles, inscribed angles, radii, and chords within a circle.
- Describe the different relationships amongst inscribed angles, radii, and the chords of a circle.
- Construct both inscribed and circumscribed circles of a triangle.
- Construct a quadrilateral inscribed in a circle and prove properties of angles for this quadrilateral.
- Construct a tangent line from a point outside a given circle to the circle.
- Determine that the length of an arc intercepted by an angle is proportional to the radius.
- Define the radian measure of an angle as a constant of proportionality.
- Find the area of a sector of circle.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Provide groups with measurement tools, such as a protractor and rulers.
- Students draw a circle with a radius of their choice. Next they draw another circle that is a dilation of the first. Students can choose any size dilation they wish
- Each group should use the same measurement tool to measure and record the radii, diameter, and circumference of their circles
- Students sketch a graph on the coordinate plane of the diameters vs. the circumference of their circles
- Next, students find the slope of the resulting line using the graph
- Groups compare graphs and slopes and discuss the following questions:
1) What do you notice about the slopes? 2) Do you think this holds true for all circles? 3) What do we call this ratio?

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Pencils

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- Protractor
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) EXPRESSION GEOMETRIC PROPERTIES WITH EQUATIONS (GPE) 5.G.GPE.1 / 5.G.GPE.2 / 5.G.GPE.3

Essential Understandings:

- Geometric figures can be represented in the coordinate plane.
- That algebraic properties (including those related to the distance between points in the coordinate plane) may be used to prove geometric relationships.
- The distance formula may be used to determine measurements related to geometric objects represented in the coordinate plane (e.g., the perimeter or area of a polygon).
- The algebraic relationship between the slopes of parallel lines and the slopes of perpendicular lines.

Essential Questions:

- What is the relationship between the slopes of parallel lines and of perpendicular lines?
- Given a polygon represented in the coordinate plane, what is its perimeter and area?
- How can geometric relationships be proven through the application of algebraic properties to geometric figures represented in the coordinate plane?

Student will be able to:

(What does mastery look like?)

- Use the coordinate plane to describe and prove the relationship between the slopes of parallel and perpendicular lines to solve geometric problems,
- Find a point on a directed line segment that partitions the segment in a given ratio.
- Use the distance formula to determine the length of a segment or side of a polygon, using this information to find either the perimeter or area of this polygon.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on the midpoint of a line segment
- Instruct students to find the midpoint of a line segment. Give each student a piece of 4-inch string licorice along with the following instructions.
 - Find the middle and lightly mark the half-way point without using a measurement tool.
 - Next, lay the licorice on graph paper finding the end point coordinates, and labeling them.
 - Connect the points to form a line segment.
 - Investigate ways to find the midpoint of the segment.
 - Write the midpoint as an ordered pair.
 - Describe how you found the midpoint of your line segment.
- Put students into groups of three or four. Give each group a worksheet with two pairs of coordinates per one graph. They will graph the pairs of points, connect the points to form a line segment, and create a formula for the midpoint. They must find the midpoint using one method they have selected by their group. If that method does not work, groups experiment to discover another that will work.
- Teacher should monitor groups and ask leading questions: 1) How did you find the midpoint, given two end points? 2) Can you write a formula for finding the midpoint using variables and equal signs? 3) Can you find the slope of a line between the two end points? 4) What is the equation of a line in slope intercept form?

Learning Activities:

- Teacher- directed class discussion on key features.
- Have students make comparisons between the slope of a line and the midpoint of a line.
- Peer teaching through group work
- Interactive technology- as assigned by instructor to support instruction-such as: interactive websites/illustrations, Study Stack – vocabulary support, etc.)- see below
- One-to-one intervention

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- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Straight edge
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) GEOMETRIC MEASUREMENT AND DIMENSION (GMD) 5.G.GMD.1 / 5.G.GMD.2 / 5.G.GMD.3

Essential Understanding:

- The formulas for circumference, area, surface area, and volume of two- and three-dimensional geometric figures can be seen as linear and other functions of the radius.

Essential Question:

- How can familiar formulas for two-and three-dimensional geometric figures be viewed as a function and/or model?
- How is the volume and/or surface area of various three-dimensional geometric objects determined?

Student will be able to:

(What does mastery look like)

- Demonstrate knowledge of formulas for volume of cylinders, cones, and spheres by applying them to and finding solutions for real-world and mathematical problems.
- Find the circumference and area of a circle using algebra.
- Find the volume for cylinders, pyramids, cones and spheres.

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- Describe (in words) how the formulas from various two-dimensional figures can be incorporated into the formulas for the volume of three-dimensional figures (e.g., the area of a circle is the base of the cylinder times the height of the cylinder).

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks:
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on finding the volume of a cylinder
- Review key vocabulary: Area, Base, Circle, Circumference, Pi, Radius, Volume
- Draw a circle on a Smart Board, label the diameter, radius, and circumference and give the diameter a numerical value
- Students use a calculator to find the area of the circle
- Draw a cylinder and write the following notes on the board:
 - A cylinder has parallel and equal sized circle as bases. To find the volume of a cylinder, multiply the area of a base by the height of the cylinder: $V=Bh$, V =volume, B = area of a base, h = height
 - Since the base of a cylinder is always a circle, substitute the formula for the area of a circle into the formula for the volume. $V=Pi(r^2)(h)$
- Next, perform a demonstration to determine the volume of a cylinder
- Math manipulates are one 2 inch diameter cylinder flower vase and one 4 inch diameter cylinder flower vase (both are 14 inches tall). Pour a cup of water in the first. Ask students how much of the water in the first vase will fill the second. Pour the same water from the first vase into the second vase. Students will see that it is not half full. Show them mathematically why it is $\frac{1}{4}$.
Vase one: $V= Pi(1^2)(12)$ Vase two: $V= Pi(2^2)(12)$. The ratio is 1:4

Learning Activities:

- Teacher- directed class discussion on key features of the lesson.
- Have students make comparison between cone volumes and cylinder volumes
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Interactive websites/illustrations, Study Stack – vocabulary support

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- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- 2-inch diameter flower vase
- 4-inch diameter flower vase
- 4 cups of water
- Calculators
- Circle and cylinder worksheets
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) MODELING WITH GEOMETRY (MG) 5.G.MG.1

Essential Understandings:

- Different geometric objects can be used to model the same or various physical phenomena and the object chosen to model the phenomena will be dependent upon how the model is to be used.
- The concept of density and how it may be applied in modeling problems involving area or volume.

Essential Questions:

- How can geometric properties and relationships be applied to solve problems that are modeled by geometric objects?
- What is density as it relates to area or volume?

Student will be able to:

- Use geometric properties and relationships in real-world applications that model a geometric object (e.g., a school track or field may be created by a rectangle with two half-circles placed at the opposite ends of the rectangle or a silo can be created by a cylinder with half of a sphere atop it).

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- Explain (in words) and describe (mathematically) how density relates to area and volume.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated quizzes and tests on class vocabulary or activities
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present an art activity for using geometric shapes to represent a human body
- Begin by visualizing the human figure as a combination of simple geometric forms: ovals, cones, cylinders, circles, etc.
- Ask students to draw circles for the shoulders, elbows and wrists. Then connect them to form the arms. The elbows are next to the waist. The wrist is next to the hip joint. Draw the shoulders larger than the head. Draw the circles for the knees slightly lower than the tips of the fingers. To make the thighs, draw a line from the outside of the oval shaped knee. Draw a line from the inside of the hip to the inside of the knee. This ensures that the thigh is drawn wide enough at the top and narrows as it goes down.
- The entire leg, from the top of the hip to the sole of the foot, is half the length of the whole human figure (Demonstrate a model on the overhead)
- When students have completed their geometric figure, they place a thin piece of paper over their drawing and create a second one with more accurate proportions, omitting the circles for the joints. They can be creative and add clothes, shoes, and hair

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

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List of Instructional Materials:

- Math manipulatives such as geometric solids
- Drawing paper
- Sketching pencils or charcoals
- Measurement tools

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

STATISTICS AND PROBABILITY (S)

INTERPRETING CATEGORICAL and QUANTITATIVE DATA (ID)

5.S.ID.1 / 5.S.ID.2 / 5.S.ID.3 / 5.S.ID.4 / 5.S.ID.5 / 5.S.ID.6 / 5.S.ID.7 / 5.S.ID.8

Essential Understandings:

- Data can be represented and interpreted in a variety of formats (dot plots, histograms, and box plots).
- Extreme data points (outliers) can skew interpretations of a set of data.
- Synthesizing information from multiple sets of data results in evidence-based interpretation.
- Statistics provide information about a population (data set) by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.
- Center and spread of a data set may be compared in multiple ways.
- Random sampling tends to produce representative samples and support valid inferences.
- Two data distributions can be compared using visual and numerical representations based upon measures of center and measures of variability to draw conclusions.
- Data in a two-way frequency table can be summarized using relative frequencies in the context of the data.
- A line of best fit can be generated for a set of data to model the relationship between two variables by hand or with technology.
- A line of best fit aims to minimize the vertical distances between the data points and the points on the line and may be used to make predictions within the proximity of the data.
- Making predictions for values within or near the data set is more reliable than for values far beyond the data set.
- Correlation does not imply causation.
- Exponential functions, like linear, can be used to model real-life situations.

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- Key features in graphs and tables shed light on relationships between two quantities.
- Differences between linear and exponential functions, thus allowing them to use the appropriate model.
- Units, scale, data displays, and levels of accuracy represented in situations.
- Functions can be created to best fit data represented on a scatter plot.
- Computations and interpretations are used to decide if differences between parameters are significant.
- A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related.
- The mean and standard deviation of a data set is used to fit a normal distribution.
- Statistics is a process of making inferences.
- Different data collection methods are appropriate for different situations and randomization relates to each.
- Functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation.
- Exponential functions can be determined from data and used to represent many real-life situations (population growth, compound interest, depreciation, etc.).
- The properties of a situation or data set determine what type of function (e.g., linear, quadratic, exponential, polynomial, rational, or logarithmic) should be used to model it.

Essential Questions:

- What is the role of statistics in real-world situations?
- When is it appropriate to question the results from a model compared to real-life situations?
- Which data collection method is best used for a specific context?
- How does randomization relate to a data collection method?
- How can two data distributions be compared?
- How can statistics be used to gain information about a sample population?
- How can a random sample be used to draw inferences of a larger population?
- How is a population mean estimated from data from a sample survey?
- When is the difference between parameters significant?
- From a scatterplot, how are two quantitative variables related?
- How is a data set fit to a normal curve?
- How do various representations of data lead to different interpretations of the data?
- When and how can extreme data points impact interpretation of data?
- Why are multiple sets of data used?
- How are center and spread of data sets described and compared?
- How is a data set represented in a two-way frequency table summarized?
- When is it appropriate to use causation or correlation?

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- How can computations and interpretations help to determine which model is appropriate in a given situation?
- What are the key features of a linear, quadratic, or exponential function in a modeling situation?
- How can a situation best be modeled by a linear, quadratic, or exponential function?
- How are units, scale, data displays, and levels of accuracy selected to appropriately represent a situation?
- How can a function that best fits the data from a scatter plot be determined?
- How can a scatter plot that is created or interpreted from data fit a function?
- What are key characteristics to identify when choosing a function to model a given situation?

Student will be able to:

- Use sampling to draw inferences about a population to show that statistics can be used to gain information by examining a sample of the population, and explain that random sampling tends to produce representative samples and support valid inferences.
- Represent and interpret data using a variety of formats (e.g., dot plots, histograms, box plots), ensuring that units, scales, data displays, and levels of accuracy represent the situation appropriately.
- Use statistics appropriately to shape data distributions and to compare the measures of central tendency (the median and mean) and spread (the interquartile range and standard deviation) of two or more different data sets.
- Interpret differences in shape, center and spread in terms of a context, accounting for possible extreme data points (outliers)
- Use the mean and standard deviation of a data set fit to a normal distribution and state whether the data set for which such a procedure is *or* is not appropriate.
- Summarize categorical data for two categories in two-way frequency table.
- Interpret relative frequencies (including joint, marginal, and conditional relative frequencies) in the context of the data, recognizing possible associations and trends in the data.
- Draw comparative inferences about two populations by informally assessing the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- Scatter plots may be used to represent data with two quantitative variables.
- Fit the best functions to the data in terms of the context of the situation, such as fitting line of best fit that suggests a linear association.
- Use technology to obtain the line of best fit and will aim to minimize the vertical distances between data points and the points on the line.
- Find the rate of change (the slope) and the intercept (a constant term) of a linear model in the context of the data.
- Use technology to find and interpret the correlation coefficient of a linear fit.

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- Compare and contrast the characteristics between a correlation and causation.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Facilitated group work/ monitoring conversations and work with rubric
- Complete and accurate group work demonstrating types of scatter plots
- Complete and accurate scatter plot from independent practice

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Introduce the concept of the scatter plot. Ask the students to recall the four different types of slope (positive, negative, zero, and undefined) and have students come up to the board to show what those types of slope look like
- Before explaining the different types of associations, have the students draw three blank scatter plots (no points) on a sheet of paper and label them Positive Association, Negative Association, and No Association. They will use M&Ms to show what they think the different types of associations look like. Then ask them to add two more blank scatter plots and label them Linear Association and Nonlinear Association. The students will again use M&Ms to show what those types of associations look like
- Review the identification of linear or nonlinear associations, as well as outliers and clusters in a scatter plot. Then show three different scatter plots on the overhead. In pairs, students identify each scatter plot as having positive, negative, or no association, whether the association is linear or nonlinear, and identify if the scatter plot has an outlier and/or cluster
- Before explaining to students how to write the equation of a line for the trend line, have students individually write down the steps of writing the equation of a line in their notes
- After explaining how students can interpret the slope and the y-intercept, give the students a sample problem that includes a scenario that can be described by a linear equation, and have the students interpret the slope and the y-intercept given what each variable represents
- During the guided practice, students will create and analyze a scatter plot for a given set of data with guidance from the teacher
- During the independent practice, students will create and analyze a scatter plot for a given set of data

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Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers
- Graph paper
- Measuring table
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

MAKING INFERENCES and JUSTIFYING CONCLUSIONS (IC)

5.S.IC.1 / 5.S.IC.2 / 5.S.IC.3 / 5.S.IC.4 / 5.S.IC.5 / 5.S.IC.6

Essential Understandings:

- Statistics can be a tool for making inferences about population versus sample parameters.
- Results from a model may or may not be consistent with real-life situations of the process.
- Different data collection methods are appropriate for different situations and randomization relates to each.
- Data from a sample survey are used to estimate a population mean.
- Real-life situations are used to decide if differences between parameters are significant.
- A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related.
- The mean and standard deviation of a data set are used to fit a normal distribution.
- Every day decisions are made based on data collection and interpretation.

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Low Adult Secondary Education (Grade Levels 7 – 8)

Essential Questions:

- How can statistics be used to understand parameters of a population versus the sample population?
- When is it appropriate to question the results from a model compared to real-life situations?
- Which data collection method is best used for a specific context?
- How does randomization relate to a data collection method?
- How is a population mean estimated from data from a sample survey?
- From a scatterplot, how are two quantitative variables related?
- How is a data set fit to a normal curve?
- How can reports or publications be evaluated based on the data presented?

Student will be able to:

(what does mastery look like)

- Uses statistics to understand the parameters of a population versus a sample population, explaining how data from a sample survey can be used to estimate a population mean.
- Use a simulation to decide if a specified model is consistent with results from a given data-generating process.
- Determine the purposes of and differences among sample surveys, experiments, and observational studies, explaining how randomization relates to the data in each.
- Use data from a randomized experiment to compare two treatments, using simulations to decide if differences between the parameters are significant.
- Explain how reports or publications can be evaluated based upon the data being presented.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate plot of given data
- Complete and accurate recording of slope, intercept, and equation of their line
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

NRS Level 5 Low Adult Secondary Education (Grade Levels 7 – 8)

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on evaluating reports based on data
- Students choose two variables that they think may have a relationship (e.g., height vs. weight)
- Students collect data either from classmates, classroom, or online
- Students create a coordinate plane with an x and y axis labeled with two chosen variables, a number scale, and title
- Students plot the given data
- Students then draw a best-fit line for their data. If the data has no correlation, they can draw a line to continue with the forthcoming parts of the activity
- After students have drawn the line, they gather the slope and intercept from their line and find the equation of the line
- A teacher-directed discussion can summarize the positive, negative, and no correlation findings, encouraging students to compare their findings with those of classmates

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutor
- Focus on articulating a clear statement on “equation relationships”
- Peer teaching through group work

List of Instructional Materials:

- Graph paper
- Rulers
- Data collection sheets
- Suggested ideas on data that can be collected

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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STATISTICS AND PROBABILITY (SP)
USING PROBABILITY TO MAKE DECISIONS (MD)

5.S.MD.1 / 5.S.MD.2 / 5.S.MD.3 / 5.S.MD.4 / 5.S.MD.5 / 5.S.MD.6 / 5.S.MD.7

Essential Understanding:

- Written descriptions, tables, graphs, and equations are useful in representing and investigating decision-making relationships in everyday life and work.

Essential Question:

- How are written descriptions, tables, graphs, and equations used in representing and investigating decision-making relationships in everyday life and work?

Student will be able to:

(what does mastery look like)

- Write descriptions with defined variables of interest that are useful in representing and investigating decision-making relationships in everyday life or work and support these descriptions with mathematical data.
- Graph a corresponding probability distribution using the same graphical displays as for data distributions.
- Find the expected value of a random variable, interpreting it as the mean of the probability distribution.
- Create a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated, finding an expected value.
- Create a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically, finding an expected value.
- Find, evaluate, and compare strategies on the basis of an expected payoff for a game of chance.
- Use probabilities to make fair decisions by drawing lots or using a random number generator.
- Use different probability concepts to analyze decisions and strategies within the context of a situation.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Facilitated group work/ monitoring conversations and work with rubric
- Completed “Practice: Expected Value” section on Khan Academy

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on random variables using selected material from <https://www.khanacademy.org/math/statistics-probability/random-variables-stats-library>
- Use the videos and problems from the “Discrete and continuous random variables and probability models” and “Expected value” sections of Khan Academy to discuss and model the content
- Students complete the “Practice: Expected Value” section and attempt the five questions, using as few hints as necessary

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutor
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts

List of Instructional Materials:

- Graph paper
- Khan Academy <https://www.khanacademy.org/math/statistics-probability/random-variables-stats-library>
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 6**

April 2017
REVISED June 2021

NRS Level 6 Math
High Adult Secondary Education (Grade Levels 9 – 12)

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NRS Level 6 Math

High Adult Secondary Education (Grade Levels 9 – 12)

NUMBER AND QUANTITY (N)

Quantities (Q)

6.N.Q.1 / 6.N.Q.2

Essential Understandings:

- Relationships can be represented quantitatively using appropriate units to solve problems in the exploration of real-world situations.
- Quantitative models can be created, used, and interpreted to solve real-life situations by using appropriate units.

Essential Questions:

- When is it advantageous to represent relationships between quantities numerically?
- Why are procedures and properties necessary when manipulating numeric expressions?
- What real world situations can be modeled by using a numerical quantity and an appropriate unit?
- What are complex numbers, and when might they appear in mathematical problems?

Student will be able to:

- Define, create, use, interpret, and represent quantitative relationships using appropriate units when solving a real-world situation.
- Find a level of accuracy appropriate to the limitations on measurement when reporting quantities.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed Advanced Fire activity and worksheet
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math Journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

This is a lesson outline helping students explore the real-world implications of functions and independent/dependent variables.

NRS Level 6 Math

High Adult Secondary Education (Grade Levels 9 – 12)

How Fast Can a Fire Advance?

1. Let students know what they will be doing and learning.
2. Teacher input is to lead a discussion to prepare students to identify independent and dependent variables in the Advanced Fire exercise.
3. Guide practice by demonstrating how to start activity on the computer. A) Setting and changing the probability that fire will spread. B) Setting a tree on fire and watching it spread.
4. Discuss the fact that Advanced Fire has multiple variables.
5. Independent practice should be done by grouping students into 3 or 4 and asking them to choose variables related to the size of the forest and the proportion of trees left standing.
6. For closure, initiate a class discussion on all findings on their chosen independent and dependent variables. They can also describe the relationships that they found between the two variables.

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Pencil and calculator
- Copy of activity and worksheet from <http://shodor.org/interactivate/lessons/AdvancedFire/>

List of Technology Resources:

- <http://shodor.org/interactivate/lessons/AdvancedFire/>
- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

NRS Level 6 Math

High Adult Secondary Education (Grade Levels 9 – 12)

NUMBER AND QUANTITY (N) The Real Number System (RN) 6.N.RN.1 / 6.N.RN.2

Essential Understandings:

- Rational expressions can be written equivalently using rational exponents.
- Properties of integer exponents may be applied to expressions with rational exponents.

Essential Questions:

- How can radical and rational exponents be written equivalently?
- How do the properties of integer exponents apply to rational exponents?

Student will be able to:

- Write equivalent rational expressions using rational exponents.
- Apply the properties of integer exponents to expressions involving radicals and rational exponents.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes
- Textbook unit reviews and/or exercises as quizzes and chapter tests

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Extend examples of relationships between exponents and radicals through interactive technology
- Use practice worksheets to develop skills of rational exponent usage
- Students will work in groups, reasoning about general statements on rational and irrational numbers
- Students record in their math journals explain and justify their classification decisions
- Students should work independently to improve their individual solutions to the assessment task

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention

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- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts

Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Mini white boards
- Poster paper, markers, scissors, large sheets of paper, glue sticks
- Calculators
- Copies of hints such as Rational and Irrational numbers and extension sheets
- *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.

List of Technology Resources:

- Kuta Software—<https://www.kutasoftware.com>
- Purple Math—www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

NRS Level 6 Math High Adult Secondary Education (Grade Levels 9 – 12)

The Complex Number System (CN)

6.N.CN.1 / 6.N.CN.2 / 6.N.CN.3 / 6.N.CN.4 / 6.N.CN.5 / 6.N.CN.6 / 6.N.CN.7 /
6.N.CN.8 / 6.N.CN.9

Essential Understandings:

- Arithmetic operations can be performed with complex numbers in standard form $(a + bi)$
- Complex numbers exist and can arise in mathematical representations of real-world situations.
- Every complex number has a conjugate and it can be used to solve expressions.
- Complex numbers are subject to the commutative, associative, and distributive properties.
- The relationship between the real and complex factors of a quadratic equation.
- There is at least one complex zero in every polynomial function of a positive degree with complex coefficients.

Essential Questions:

- What are complex numbers, and when might they appear in mathematical expressions?
- Which arithmetic operation can be used to create an appropriate complex number to model a given situation?
- How can complex numbers be represented in the rectangular and polar coordinate systems?
- What changes are made to a complex number to find its conjugate?
- Using the relationship $i^2 = -1$, how can the commutative, associative, and distributive properties be used in the arithmetic operations of complex numbers?
- How can complex numbers be used to solve a quadratic equation with real coefficients?
- What is the relationship between the real and complex factors of a quadratic equation and the x-intercepts of a graph of the quadratic?

Student will be able to:

- State what a complex number is, when it might appear in a mathematical expression, and which arithmetic operations can be used to create a complex number.
- Write a complex number in $a + bi$ form and represent this number in the rectangular and polar coordinate system.
- Show how the commutative, associative and distributive properties are used in the arithmetic operations of complex numbers by using the relationship $i^2 = -1$.

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- Find the conjugate of a complex number written in $a + bi$ form and explain how it can be used to solve expressions
- Explain why the rectangular and polar forms of a given complex number represents the same number.
- Represent addition, subtraction, multiplication and the conjugate of complex numbers geometrically on a complex plane, using properties of the representation for computation.
- Find the distance between numbers in the complex plane as the modulus of the difference.
- Find the midpoint of a segment in the complex plane as the average of numbers at its endpoints.
- Use complex numbers to solve quadratic equations with real coefficients.
- Describe the relationships between the real and complex factors of a quadratic equation in terms of the x-intercepts of a graph or the zeros of the function.
- Use the Fundamental Theorem of Algebra to show that it is true for quadratic polynomials.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed worksheet and activity from “Imagining a New Number Learning Task”
- Plato Learning Environment tests-monitor management system by objective and NRS level.

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

“Imagining a New Number Learning Task” found at lhsblogs.typepad.com

Activating Strategies: (Learners Mentally Active)
Share the historical story of “ i ” from “Imagining a New Number Learning Task”
Acceleration/Previewing: (Key Vocabulary)
Imaginary form, complex number, “ i ”, standard form, pure imaginary number, complex conjugates, and complex number plane, absolute value of a complex number

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Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)
Walk through the “Imagining a New Number Learning Task.” Insert the graphic organizers on pages as needed. Use small groups and collaborative pairs to complete the task, including guided practice on teacher-made practice sheets. At the end of each lesson, groups share with the class.
Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)
What exactly is the absolute value of any number? How does the definition of absolute value apply to the complex plane? Does the definition of absolute value also work with real numbers? 22ba+ How do you write a real number as a complex number?
Extending/Refining Strategies:
Solve: $3x^2 - \sqrt{2}x + 2 = 0$ and $\sqrt{2}x^2 - 6x + \sqrt{8} = 0$

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Graph paper
- Handouts from “Imagining a New Number” learning activity - lhsblogs.typepad.com

List of Technology Resources:

- Purple Math www.purplemath.com
- Cool Math www.coolmath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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NUMBER AND QUANTITY (N) Vectors and Matrix Quantities (VM)

6.N.VM.1 / 6.N.VM.2 / 6.N.VM.3 / 6.N.VM.4 / 6.N.VM.5 / 6.N.VM.6 / 6.N.VM.7 /
6.N.VM.8 / 6.N.VM.9 / 6.N.VM.10 / 6.N.VM.11 / 6.N.VM.12

Essential Understandings:

- Directed line segments and appropriate symbols are used to represent and solve velocity and other quantities that represent a vector.
- Vector components are found by subtracting the initial point from the coordinates of a terminal point
- The operations of addition, subtraction and multiplication can be applied to vectors.
- Matrices can be used to represent and manipulate data.
- The operations of addition, subtraction and multiplication can be applied to matrices of appropriate dimensions.
- Knowledge of the zero and identity matrices, as well as the determinant, can be applied in matrix addition and multiplication.
- New matrices can be produced by multiplying matrices by scalars.
- Matrices can be used in the transformation(s) of a vector.

Essential Questions:

- What is the purpose of recognizing and writing vectors quantities, having both a magnitude and direction?
- How can vectors represent vector quantities as directed line segments?
- How can the components of a vector be found?
- How can vectors involving velocity and other quantities be represented?
- What would be the result be if vectors were added, subtracted and/or multiplied?
- How are scalars used in matrix multiplication?
- Based upon what is known about matrices, how would the addition, subtraction and multiplication of two matrices be performed and explained?
- What is the role of the zero and identity matrices in matrix addition and multiplication?
- When is the determinant of a square matrix nonzero?
- How can matrices be used in the transformation of vectors?

Student will be able to:

- Write vector quantities having both a magnitude and a direction.
- Solve velocity and quantities representing vectors with directed line segments and appropriate symbols.
- Find vector components by subtracting the initial point from the coordinates of a terminal point.
- Use either the end-to-end, component-wise, or the parallelogram rule to add and subtract vectors.

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- Find the magnitude and direction form to determine the magnitude and direction of the sum of two vectors.
- Represent vector subtraction graphically by connecting tips in the appropriate order, performing vector subtraction component-wise.
- Define vector subtraction of $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w , having the same magnitude as w , but pointing in the opposite direction.
- Multiply a vector by a scalar.
- Represent scalar multiplication graphically, scaling vectors and possibly reversing the direction.
- Find the magnitude of a scalar multiple of cv using $||cv|| = |c|v$, knowing the direction of cv can be either along v (for $c > 0$) or against v (for $c < 0$).
- Use matrices to represent and manipulate data.
- Use scalars in matrix multiplication.
- Add, subtract and multiply matrices.
- Use scalars in matrix multiplication.
- Uses the zero and identity matrix in matrix addition and multiplication.
- Use a determinant of a square matrix, which is zero, if and only if the matrix has a multiplicative inverse.
- Multiply a vector by a one column matrix by a matrix of suitable dimensions to produce another vector, using matrices in the transformation of vectors.
- Use a 2×2 matrices as transformations of a plane, interpreting the absolute value of the determinant in terms of area.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed Purple Math matrix and vector activities and problems
- Optional: completed “Vector Voyage!” assignment (including Worksheets 1 and 2)
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson demonstrating how to represent vectors geometrically, how to scale vectors, and how to combine vectors by addition. Encourage students to explore how vectors can be used to solve applied problems

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- Use material from PurpleMath.com relating to vectors and matrices to introduce the following concepts:
 - Matrix Definitions
 - Adding and Subtracting Matrices
 - Matrix Multiplication
 - Matrix Row Operations
- Explain that vectors and vector operations are used extensively in navigation on water and air. Display on overhead the website Teach Engineering and visit the lesson “Vector Voyage!” Cover the introductory material as a group and discuss how vectors can be represented geometrically with directed line segments and how vectors and scalar multiples of vectors can be used to model navigation routes
- Assign this “Vector Voyage!” lesson as group work or an extension/homework activity as appropriate

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Purplemath.com
- Teach Engineering.org

List of Technology Resources:

- Purple Math Advanced Algebra Topics www.purplemath.com
- Teach Engineering “Vector Voyage!” Activity https://www.teachengineering.org/activities/view/cub_navigation_lesson02_activity1
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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High Adult Secondary Education (Grade Levels 9 – 12)

ALGEBRA (A)
Seeing Structure in Expressions (SSE)
6.A.SSE.1 / 6.A.SSE.2 / 6.A.SSE.3

Essential Understandings:

- The different parts of expressions can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform expressions.
- Exponential expressions have equivalent forms that can reveal new information to aid in solving problems.
- The factors of a quadratic expression/equation can be used to reveal the zeros of the quadratic.
- There are several ways to solve a quadratic equation (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the equation.
- Quadratic expressions have equivalent forms that can reveal new information to aid in solving problems.
- Equivalent forms of an expression can be found, dependent on how the expression is used.
- Geometric sequences have a domain of integers with equal factors (constant ratios).
- Arithmetic sequences have equal intervals (common difference).
- Geometric sequences can be represented by both recursive and explicit formulas.
- Expressions represent a quantity in terms of its context.
- Expressions have equivalent forms that can reveal new information to aid in solving problems.
- Exponential expressions, like linear expressions, can be used to model real-life situations.
- Differences between linear and exponential expressions allow students to use the appropriate model.

Essential Questions:

- How are expressions used to solve real world problems?
- When is it advantageous to represent relationships between quantities symbolically? Numerically?
- Why are procedures and properties necessary when manipulating numeric or algebraic expressions?
- How can the structure of expressions help determine a solution strategy?

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- What new information will be revealed if an expression is written in a different but equivalent form?
- What do the key features of an exponential or linear expression represent in a modeling situation?
- How is it determined if a situation is best modeled by an exponential or linear expression?
- What does completing the square reveal about a quadratic expression?
- What do the factors of a quadratic reveal about the expression?

Student will be able to:

- Create an equivalent form of a quadratic expression by completing the square, which will reveal the maximum or minimum of the function it defines.
- Write an expression in a different but equivalent form to obtain new information for an exponential function.
- Use the formula to solve a geometric series to solve real-world situations.
- Derive the formula for the sum of a finite geometric series when the common ratio is not 1.
- Write equivalent expressions using either the properties of integer exponents or the sum and difference of squares.
- Factor a quadratic expression to reveal its zeros of the function it defines and explain the meaning of the zeros.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate solutions to five problems from Laws of Exponents game
- Complete and accurate solutions to “Completing the Square” material

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Certain rules apply to exponents, and students will learn and use these rules to simplify algebraic expressions and build upon them in future lessons
- Review material from PurpleMath.com section “Exponents: Basic Rules”
- Direct practice problems to the whole class, then to partners, and then have students complete some problems independently
- Tell students they will practice the laws of exponents with a small group game. Students take turns until everyone in the group has gotten at least five turns. How to play the game:

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- Divide students into groups of 3. Have one member from each group get a spinner, a paper clip, and a penny. (Show students how to use their pencil and paper clip as a spinner)
 - Students take turns spinning the spinner to get the first x and exponent. Then they flip the coin to determine the function of division or multiplication. Then they spin the spinner again to get a second x and exponent
 - Give each member of the group a piece of paper and pencil to record their work and answers – to be submitted to teacher after the game
-
- As a whole group, review the material on PurpleMath.com section “Completing the Square: Solving Quadratic Equations”
 - Have students complete a few of the examples as group work to demonstrate competency

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Exponents: Basic Rules material from PurpleMath.com
- Laws of Exponent Game instructions
- Spinners
- Paper clips
- Pennies
- “Completing the Square: Solving Quadratic Equations” material from PurpleMath.com

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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ALGEBRA (A)

Arithmetic with Polynomials and Rational (APR)

6.A.APR.1 / 6.A.APR.2 / 6.A.APR.3 / 6.A.APR.4 / 6.A.APR.5 / 6.A.APR.6 /
6.A.APR.7

Essential Understandings:

- Polynomial expressions can be added, subtracted, and multiplied to produce new polynomials.
- Applied problems using quadratic expressions can be answered by either solving or re-writing the quadratic expression in a more useful form (factoring to find the zeroes, or completing the square to find the maximum or minimum, for instance).
- There are several ways to solve a quadratic expression (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the expression.
- The quadratic formula is derived from the process of completing the square.
- Quadratic expressions have equivalent forms that can reveal new information to aid in solving problems.
- The Remainder Theorem can be used to determine roots of polynomials
- Polynomial and rational expressions can be added, subtracted, and multiplied to produce new polynomials.
- The factors of a quadratic can be used to reveal the zeroes of the quadratic.
- The process of completing the square can be used to reveal the vertex of the graph of a quadratic expression (and consequently the minimum or maximum of the function).
- The degree of a polynomial helps to determine the end behavior of its graph.
- The zeroes of each other of a polynomial expression determine the x -intercepts of its graph.
- Graphs of rational expressions are often discontinuous, due to values that are not in the domain of the expression.
- The long division algorithm for polynomials can be used to determine horizontal or oblique asymptotes of rational expressions.

Essential Questions:

- How do the arithmetic operations on numbers extend to polynomials?
- How can a quadratic expression be simplified?
- How do the factors of a quadratic determine the x -intercepts of the graph and vice versa?
- When a polynomial $p(x)$ is divided by $x-a$, how can its remainder be found?
- How do the arithmetic operations on numbers extend to polynomials?

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- What do the factors of a quadratic reveal about the function?
- What does completing the square reveal about a quadratic function?
- What is the graph of a quadratic function? What are its properties?
- How can a rational expression be simplified?

Students will be able to:

- Add, subtract and multiply polynomial expressions to produce new polynomials.
- Use the Remainder Theorem to determine roots of a polynomial.
- Factor or complete the square to solve a quadratic expression for the zeros (found when factoring), and the maxima or the minima (found when completing the square).
- Find the factors of a quadratic function and determine how these factors relate to the x-intercepts (the zeros) of the graph or vice versa.
- Prove polynomial identities and use them to describe numerical relationships (e.g., generate Pythagorean triples).
- Use the Binomial Theorem for expansion of $(x + y)^n$ in powers of x and y for a positive integer n, with coefficients determined by Pascal's Triangle.
- Rewrite simple rational expressions in different forms, using inspection, long division, or technology.
- Prove that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression.
- Add, subtract, multiply, and divide rational expressions.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Independent notes and work on Remainder Theorem
- Group and independent work to PurpleMath.com problems for Rational Expressions and Binomial Theorem
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

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- Present a lesson for students to successfully learn and apply the Remainder Theorem for a polynomial and a number
- Explain why the Remainder Theorem is useful for evaluating polynomials at a given value of x . Students do not necessarily have to understand it as a proof; they need to understand how to use the Theorem.
- Explain that The Remainder Theorem starts with an unnamed polynomial $p(x)$ where “ $p(x)$ ” means some polynomial p whose variable is “ x ”. The Theorem works by dividing that polynomial by some linear factor $x-a$, where “ a ” is a number.
- Start with a concrete example of long polynomial division: $p(x) = x^3 - 7x - 6$ and divide by the linear factor, $x - 4$ ($a=4$).

$$\begin{array}{r}
 \underline{x^2 + 4x + 9} \\
 x - 4 \overline{) x^3 + 0x^2 - 7x - 6} \\
 \underline{-x^3 + 4x^2} \\
 4x^2 - 7x - 6 \\
 \underline{-4x^2 + 16x} \\
 9x - 6 \\
 \underline{-9x + 36} \\
 30
 \end{array}$$

- This produces a quotient of $q(x) = x^2 + 4x + 9$ on top, with a remainder of $r(x) = 30$.
Remind students that when there is no number in front of x , the degree on x is understood as “1.” In polynomial terms, we are dividing by a linear factor, so the remainder must be a constant value, since the degree on x is “1”.
- The Remainder Theorem then points out the connection between division and multiplication. For example, since $12 \div 3 = 4$, then $4 \times 3 = 12$. If you get a remainder, you do the multiplication and then add the remainder back in. For example, $13 \div 5 = 2 \text{ R } 3$, then $13 = 5 \times 2 + 3$. This process works the same way with polynomials. Since $(x^3 - 7x - 6) \div (x - 4) = x^2 + 4x + 9$ with a remainder of 30, then $x^3 - 7x - 6 = (x - 4)(x^2 + 4x + 9) + 30$.
- Present a lesson to students on rational expressions and how to rewrite them using material from the PurpleMath.com Advanced Algebra Topics
- Walk through the lesson titled “Rational Expressions: Simplifying” and ask students to complete some of the sample problems in pairs or independently
- Assign the lesson from PurpleMath.com titled “Binomial Theorem: Formulas” as an extension activity or homework as appropriate

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work

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- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

(core and supplemental)

- PurpleMath.com Rational Expressions and Binomial Theorem lessons

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

ALGEBRA (A)

Creating Equations (CED)

6.A.CED.1 / 6.A.CED.2 / 6.A.CED.3 / 6.A.CED.4

Essential Understandings:

- Linear models can be created, used, and interpreted for real-life situations.
- Real world situations can be modeled by systems of linear equations.
- A system of equations can have no, one, or infinitely many solutions.
- Solutions of systems of inequalities are ordered pairs that satisfy all equations as well as inequalities that are often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Multiple methods may be used to solve a system of equation or inequalities.
- Functions can be created to best fit data represented on various models.
- Polynomial functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation.
- Systems can be solved graphically, algebraically or from a table.

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Essential Questions:

- What real world situations can be modeled by a linear relationship?
- How can technology help to determine whether a linear model is appropriate in a given situation?
- How can systems of linear equations or inequalities be used to model real world situations?
- How can the solution(s) of a system be represented and interpreted?
- What processes may be used to solve a system of equations or inequalities?
- How can a linear function be found that best fits data from various models?
- What are the different methods that can be used to find the solutions of a system of equations?
- When changes are made to an equation, what changes are made to the graph?
- What new information will be revealed if a formula is written in a different but equivalent form?
- How can the solution(s) of a system be represented and interpreted?

Student will be able to:

- Create and interpret various types of equations and inequalities in one variable, using them to solve problems.
- Create and interpret various types of equations in two or more variables, describing the relationship of the two quantities being represented and determining whether the solution(s) are viable or nonviable for the modeling context.
- Graph systems of either equations or inequalities on a coordinate axes, properly labeling and scaling the axes.
- Graph systems of equations or inequalities on a coordinate axes, properly labeling and scaling the axes.
- Rewrite formulas or literal equations to highlight a quantity of interest.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric
- Plato Learning Environment tests-monitor management system by objective and NRS level

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Students survey different word problems that involve writing an equation with one variable. Then they write their own word problem that can be solved by a one- step equation. Students then perform the same procedure with two-step equations
- Students then experiment with different methods of solving systems of equations
- Demonstrate graphing equations and inequalities on a coordinate axes.
- Students experiment with graphing sample equations and inequalities in small groups
- Teacher- directed class discussion on key features of the quadratic graph
- Have students make comparisons between quadratic graphs and linear graphs

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring through group work
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts

Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Straight edge
- Pencils
- Formula worksheet from GED® testing service:
<http://www.gedtestingservice.com/uploads/files/0756c16704434ff71e43c8117a5fa738.pdf>
- *Possible Texts*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com

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- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

ALGEBRA (A)

Reasoning with Equations and Inequalities (REI)

6.A.REI.1 / 6.A.REI.2 / 6.A.REI.3 / 6.A.REI.4 / 6.A.REI.5 / 6.A.REI.6 / 6.A.REI.7

Essential Understandings:

- The different parts of an expression, simple rational and radical equations, inverse matrices (if it exists), and inequalities can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities in one and two variables.
- Equivalent forms of an expression can be found, dependent on how the expression is used.
- Real world situations can be modeled by systems of linear equations having no, one, or infinitely many solutions.
- Real world situations of systems of inequalities are ordered pairs that satisfy all inequalities, often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Discrete and continuous functions of the first and second degree have properties that appear differently when graphed.
- Exponential expressions represent a quantity in terms of its context and have equivalent forms that can reveal new information to aid in solving problems.
- Exponential functions can be determined from data and used to represent many real-life situations (e.g., population growth, compound interest, depreciation, etc.) by a table, graph, verbal description, or through the use of technology. Each representation can be transferred to another representation.
- Logarithms can be used to solve the exponential equations modeling and can be useful to represent numbers that are very large or that vary greatly and are used to describe real-world situations (e.g., Richter scale, Decibels, pH scale, etc.).

Essential Questions:

- How are various equations, system, and inequalities used to solve real world problems?

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- When is it advantageous to represent relationships between quantities symbolically? Numerically? Graphically?
- How can the structure of linear, polynomial, rational, absolute value, exponential, logarithmic, expressions, equations, inequalities help determine a solution strategy?
- How can the solution(s) of a system be represented and interpreted?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- How can technology help to determine whether a linear, polynomial, rational, absolute value, exponential, or logarithmic model is appropriate in a given situation?

Student will be able to:

- Find the solution of a simple rational and radical equation in one variable, giving examples showing how extraneous solutions may arise.
- Find the solution of a simple linear equation and a quadratic equation in two variables algebraically and graphically.
- Write a system of linear equations as a single matrix equation in a vector variable.
- Write and find the inverse of a matrix, using it to solve systems of linear equations. (Note: Technology may be used.)
- Find the x-coordinates of the points where the graphs of equations $y = f(x)$ and $y = g(x)$ intersect and explain why these are the solutions of the equation $f(x) = g(x)$.
- Using technology, tables of values, or successive approximations to find the exact and approximate solutions for functions which can be linear, polynomial, rational, absolute value, exponential, or logarithmic.
- Graph the solutions of a linear inequality in two variables as a half-plane.
- Graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate notes and work on radical equations
- Complete and accurate solutions to Khan Academy problems on linear systems and linear inequalities
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

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- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on Solving Radical Equations
- Review key terms: Radical equation, Radican
- Ask students: What do you know about solving equations? Let's look at an equation with a variable as a radicand: $\sqrt{x} = 6$
- For what value of x would you substitute to make this equation a true statement? Examine the algebra to solve this problem

$\sqrt{x} = 6$ $\sqrt{x} = \sqrt{36}$ $x = 36$	$\sqrt{x} = 6$ $(\sqrt{x})^2 = 6^2$ $x = 36$	$\sqrt{x} = 6$ $x^{1/2} = 6$ $(x^{1/2})^2 = (6)^2$ $x = 36$
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- Additional examples to be done as group or independent work:
 1) $\sqrt{x+5} = 11$ 2) $\sqrt{x-4} = 7$ 3) $\sqrt{x-3} = 0$
- Students can also be taught to solve quantities such as $x-5$ under the entire radicand
- Present a lesson on linear equations using material from Khan Academy including: "Solving Linear Systems with Matrix Equations" and "Solving and Graphing Linear Inequalities"
- Students complete some of the sample problems in pairs or independently

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Overhead projector
- Khan Academy material on linear systems and linear inequalities

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List of Technology Resources:

- Khan Academy Linear Systems - <https://www.khanacademy.org/math/precalculus/precalc-matrices/solving-equations-with-inverse-matrices/v/solving-matrix-equation>
- Khan Academy Linear Inequalities - <https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-one-step-inequalities/v/solving-and-graphing-linear-inequalities-in-two-variables-1>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

FUNCTIONS (F)

Interpreting Functions (IF)

6.F.IF.1 / 6.F.IF.2 / 6.F.IF.3 / 6.F.IF.4 / 6.F.IF.5 / 6.F.IF.6 / 6.F.IF.7 / 6.F.IF.8 /
6.F.IF.9 / 6.F.IF.10

Essential Understandings:

- Functions and its notation have exactly one output for each input and can be defined explicitly or recursively.
- Given a particular representation (such as an equation) of a function, other representations (such as graphs or tables) can be generated and explored.
- Functions (square root, cube root, piecewise, polynomial, rational, exponential, and logarithmic) exhibit key features that can be identified and used to compare functions or to determine solutions to real world experiences.
- Average rate of change can be calculated, estimated and/or interpreted from multiple representations of a function.
- Sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.
- Graphs of rational functions are often discontinuous, due to values that are not in the domain of the function.
- That $\log_b y = x$ is another way of expressing $b^x = y$ and that this logarithmic expression can be used to determine the solution of an equation where the unknown is in the exponent.
- The graphs of various functions have key features, including domain, intercepts, where the function is increasing or decreasing (positive or negative) behavior, relative maximums and minimums, symmetries, and end behavior.
- The zeros of each factor of a polynomial determine the x-intercepts of its graph.
- Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).

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Essential Questions:

- What are various representations of a function and how can they be interpreted?
- How are key features of a function identified and explained in relation to the context?
- How are functions and their properties including the increasing or decreasing (positive or negative) behavior, relative maximums and minimums, symmetries, and end behavior compared?
- What determines the type of sequence or function is represented in a real-world situation?
- What are the different ways an exponential function be represented?
- What are the key features of a function or graph and how is it best modeled?
- How is the domain of a rational function related to its graph?
- How can rewriting the equation of a rational function (using long division of polynomials) give further information about its graph?
- What are the key features of a linear or quadratic function? Slope? Intercepts? Maxima? Minima?

Student will be able to:

- Define the domain (from one function set) and range (from another function set), assigning each element of the domain to exactly one element of the range.
- Use $f(x)$ to denote the output of function f corresponding to the input or domain and the graph of function f is the graph of $f(x)$.
- Use appropriate function notation, evaluating functions for inputs in their domains and interpreting statements that use function notation in terms of a context.
- Determine and use the appropriate sequence or function (sometimes recursive) that best models a real world situation.
- Model a relationship between functional quantities, interpreting key features of the function graphs and tables in terms of the quantities.
- Draw a graph expressed symbolically and show key features of the graph, by hand in simple cases and/or using technology for more complicated cases.
- Compare two functions each represented either algebraically, graphically, given tables or by a verbal description in terms of their domains, their intercepts, their increasing and decreasing behavior, their relative maximums and minimums, their symmetries, and their end behaviors.
- Calculate, estimate, or interrupt the average rate of change from multiple representations of a function.
- Represent, interpret, and graph various representations of a function (e.g., the square root function, the cubic function, the piecewise function, the polynomial function, the rational, exponential and logarithmic function) by

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the process of factoring and completing the square to show zeros, extreme values, and symmetry of the graph, interpreting these concepts in terms of a context.

- Graph linear functions showing intercepts.
- Graph quadratic functions showing intercepts and either maxima or minima.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed and accurate Super Bowl ad activity sheet
Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

Super Bowl commercial 2014: Still increasing!

- The table below shows the cost of a 30-second Super Bowl TV advertisement each year since 1967. Create a scatter plot of the data on the grid below. When creating a scatter plot, do not connect the data points with lines as with a line graph. Be careful setting up the scale for the y-axis.
 1. How has the cost of a 30 second Super Bowl commercial grown over time? Describe this in detail. Has it increased or decreased? Has it grown at a constant rate of change?
 2. Try to draw a smooth curve that models the cost of an ad over time. It should go through many of the data points, but may not go through all of the data points. It should be a good model of how the cost is growing over time and be usable to make future predictions.
 3. As a group, try to find the quotient between each year of Super Bowl ads. Write these quotients to the right of the table on the first page. You should put the values to the right and between the two years that you computed. These values represent the number that we multiply by to get to the following year's cost of a Super Bowl ad.
 4. What does the typical quotient look like? Can you find an average? This average value represents a rough estimate of what we multiply by to get future ad costs.
- When we multiply by this value over and over again to find the next ad cost, we refer to it as the **growth factor**. If you found that the typical

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quotient is 1.12, this means typically the Super Bowl cost increases by 12% per year and that our average growth factor is roughly 1.12.

- Use your typical growth factor to find the potential cost of a 30-second ad during the next three Super Bowls. For example, if your typical growth factor is 1.12, then multiply the last Super Bowl cost by 1.12 to find the potential cost of an ad during Super Bowl 48. Repeat this process until you have potential ad costs for Super Bowls 49 and 50.
- Graph your potential Super Bowl ad costs for these three Super Bowls on your graph. Do they seem to fit in with the trend?
- **Note to students:** You may notice that the cost of a Super Bowl ad has not grown linearly. The cost may have grown somewhat exponentially. This means that we can multiply by the same value year after year to find the cost of the following year's ad. To determine whether this is true, simply divide an ad cost by the cost from the year before.

Bowl number	Cost of 30-second ad										
1	\$40,000										
2	\$54,000										
3	\$67,500										
4	\$78,200										
5	\$72,000										
6	\$86,000										
7	\$103,500										
8	\$107,000										
9	\$110,000										
10	\$125,000										
11	\$162,000										
12	\$185,000										
13	\$222,000										
14	\$275,000										
15	\$324,300										
16	\$345,000										
17	\$400,000										
18	\$450,000										
19	\$500,000										
20	\$550,000										
21	\$575,000										
22	\$600,000										
23	\$675,000										
24	\$700,000										
25	\$800,000										
26	\$800,000										
27	\$850,000										

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28	\$900,000	
29	\$1,000,000	
30	\$1,100,000	
31	\$1,200,000	
32	\$1,300,000	
33	\$1,600,000	
34	\$2,100,000	
35	\$2,050,000	
36	\$1,900,000	
37	\$2,100,000	
38	\$2,250,000	
39	\$2,400,000	
40	\$2,500,000	
41	\$2,600,000	
42	\$2,700,000	
43	\$3,000,000	
44	\$2,800,000	
45	\$3,100,000	
46	\$3,500,000	

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- Think out loud (demonstrate how to think about a problem)
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Super Bowl Activity sheet
- Overhead projector
- Rulers
- Calculators

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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FUNCTIONS (F)

Building Functions (BF)

6.F.BF.1 / 6.F.BF.2 / 6.F.BF.3 / 6.F.BF.4 / 6.F.BF.5

Essential Understandings:

- Functions with a domain that are a subset of the integers and can be identified by the constant difference between consecutive terms (arithmetic sequences).
- Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line.
- Geometric sequences can be represented by both recursive and explicit formulas.
- Units, scales, data displays, and levels of accuracy are represented in real-world situations.
- Find, understand, and solve the inverse and composite relationship of functions.

Essential Questions:

- What is an arithmetic or geometric sequence and how does it relate to a function?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- Which type of arithmetic or geometric sequence or function models a situation?
- How do you choose units, scale, data displays and levels of accuracy to appropriately represent a situation?
- How can the inverse and composite relationship of a function be used in a real-world situation?

Student will be able to:

- Use arithmetic operations to combine standard functions to create composite functions.
- Write arithmetic or geometric sequences both recursively or with an explicit formula to model a situation, translating between these two forms.
- Find the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(k(x))$ and $f(x + k)$ for specific values of a positive and negative k .
- Find the value of k given the graphs, explaining the effects on the graph using technology.
- Find inverse functions, solving an equation of the form $f(x) = c$ for a simple function that has an inverse and rewriting an inverse expression.
- Verify the composition that one function is the inverse of another.
- Read values of an inverse function from a graph or a table, given that the function has an inverse.

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- Create an invertible function from a non-invertible function by restricting the domain.
- Find the inverse relationship between an exponential and logarithmic function, using their inverse relationship to solve problems.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed practice problems from Khan Academy lessons on “Composing Functions” and “Manipulating Functions”
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals
- Evaluate level of group participation

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on composite functions using material from Khan Academy section “Composing Functions”
- Explain the concept and examples to students (partly through the videos) and ask pairs of students to work the sample and challenge problems and share answers for class review and discussion
- Present a lesson on finding the inverse of a function using material from Khan Academy unit “Manipulating Functions”
- Through a combination of whole-group teaching (including the videos) and small group work, have students complete all sections of the Functions unit: “Intro to Inverse Functions,” the practice problems for the intro, “Inputs and Outputs of Inverse Functions,” “Graphing the Inverse of a Linear Function,” and the Practice section for evaluating inverse functions

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)

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- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Overhead projector
- Notebooks
- Graph paper
- Graphing calculators
- Khan Academy material for “Composing Functions” and “Manipulating Functions”

List of Technology Resources:

- Khan Academy - <https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition> and <https://www.khanacademy.org/math/algebra2/manipulating-functions/introduction-to-inverses-of-functions/v/introduction-to-function-inverses>
- Composite functions - <http://www.purplemath.com/modules/fcncomp3.htm>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

FUNCTIONS (F)

Linear, Quadratic, and Exponential Models (LE)

6.F.LE.1 / 6.F.LE.2 / 6.F.LE.3 / 6.F.LE.4

Essential Understandings:

- Discrete and continuous functions have properties that appear differently when graphed.
- Arithmetic and geometric sequences that have a domain of integers, but arithmetic sequences have equal intervals (common difference) and geometric sequences have equal factors (constant ratio).
- Arithmetic and geometric sequences can be represented by both recursive and explicit formulas.
- Differences between linear and exponential functions allow these functions to be used as an appropriate model.
- Use graphs and tables to recognize a situation where a constant grows or decays by a constant percent rate.

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- Arithmetic and geometric sequences have a domain of integers, but arithmetic sequences have equal intervals (common difference) and geometric sequences have equal factors (constant ratio).
- Interpret the parameters in a linear or exponential function in terms of a real-world context (e.g., compounding returns or investment goals).

Essential Questions:

- What type of linear, quadratic or exponential function is best to model a given situation?
- How can you decide what type of sequence or function is represented?
- What are the different ways you can represent an exponential function?
- How do you create an appropriate function to model data or situations given within context?
- What new information will be revealed if this equation is written in a different but equivalent form?
- What are the different ways an exponential or linear function can be compared?
- How can the parameters of a linear or exponential function be interpreted?
- How is it determined when a situation is best modeled by an exponential or linear function?

Student will be able to:

- Create a linear or exponential function or an arithmetic or geometric sequence, given a graph, a verbal description or an input-output table, transferring easily between each of these representations to obtain a correct solution.
- State the differences between a linear and exponential function and determine which function best models a particular situation.
- Prove linear functions grow by an equal difference over equal intervals.
- Prove exponential functions grow by equal factors over equal intervals.
- Create graphs and tables to determine whether the constant grows or decays and by what constant percentage rate.
- Write a logarithmic equation for exponential models, evaluating the logarithm using technology.
- Interprets and states the parameters of either a linear or exponential function in terms of its real-world context.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes
- Chapter/unit reviews/tests from core texts
- Completed independent assignments - graded
- Facilitated group work/ monitoring conversations and work with rubric
- Complete and accurate solutions to practice problems from Khan Academy “Introduction to Exponential Functions” unit and Purple Math “Exponential Functions” unit

Other Evidence:

- Math journals
- Evaluate level of participation in group work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on functions with a skill focus on constructing a table of values and/or frequency table
- Review key vocabulary: exponential function, linear function, geometric sequence, input, output
- Start with illustrations of the application of exponential growth (For example: The population of Chocolate Town in 2012 was estimated to be 15,000 people with an annual rate of increase or growth of about 3%.
 - a) What is the growth factor for Chocolate Town after one year? $15,000 + 0.03(15,000) = 15,000(1.03)$. The growth factor is 1.03.
 - b) Write an equation to model the future growth.
 $y = ab^x$ $a = 15,000$ and $b = 1.03$
 - c) Use the estimation to estimate the population in 2016 $y = 15,000(1.03)^4$
- Explain that the growth factor is greater than 1
- Present a lesson on linear and exponential functions using material from Khan Academy section “Introduction to Exponential Functions”
- Explain the concept and examples to students (partly through the videos) and ask pairs of students to work the sample and challenge problems and share answers for class review and discussion
- Present a lesson on exponential functions using material from PurpleMath.com unit “Exponential Functions”
- Through a combination of whole-group teaching and small group work, have students complete all five sections of the Exponential functions unit.

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Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Focus on articulating a clear statement on “exponential relationships”
- Teacher-directed class discussion on key features of an exponential graph.
- Have students make comparisons between quadratic graphs and linear graphs
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Graph paper
- Rulers
- PurpleMath.com unit on Exponential Functions (5 pages)
- Khan Academy videos and practice problems on Exponential Functions (2 videos, 2 sets of practice)
- Geometric manipulatives
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Purple Math — <http://www.purplemath.com/modules/expofcns.htm>
- Khan Academy - <https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/comparing-exponential-and-polynomial-functions/v/linear-exponential-models>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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FUNCTIONS (F)

Trigonometric Functions (TF)

6.F.TF.1 / 6.F.TF.2 / 6.F.TF.3 / 6.F.TF.4 / 6.F.TF.5 / 6.F.TF.6 / 6.F.TF.7 / 6.F.TF.8
/ 6.F.TF.9

Essential Understandings:

- The unit circle allows all real numbers to work in trigonometric functions.
- Pythagorean identities can be proven and used to solve problems with specified context.
- Key features in a unit circle shed light on the relationships between two quantities.
- Trigonometric functions can be represented by a table, graph, verbal description or equation, and each representation can be transferred to another representation.
- Specific transformations occur to trigonometric functions based on a value k and its manipulation to the function.
- The trigonometric functions $\sin(x)$, $\cos(x)$, or $\tan(x)$ can be used to model real-life situations that exhibit periodic behavior.
- Changing parameters such as amplitude, period, and midline of a function will alter its graph and that these parameters are related to the context or phenomena being modeled.
- The trigonometric functions of sine, cosine, and tangent can be used to solve sum or difference problems.
- Using technology, evaluate and interpret inverse functions to solve trigonometric equations arising in real-world situations.
- Use the unit circle to explain symmetry with odd and even and periodicity of trigonometric functions.

Essential Questions:

- How can the unit circle be read and interpreted using radians?
- How does the Pythagorean theorem and the unit circle relate to the identity $\sin^2(x) + \cos^2(x) = 1$?
- What do the key features or characteristics of a trigonometric function represent?
- What are the different ways a trigonometric function can be represented?
- What transformations can occur to a trigonometric function/graph?
- How can the graphs of trigonometric functions be modified to best fit the situations being modeled?
- How do factors such as amplitude, period, midline, and horizontal shift affect these functions and relate to the phenomena being modeled?
- How can the trigonometric functions of sine, cosine, and tangent be used to solve sum or difference problems?
- How can technology be used to evaluate and interpret inverse functions to solve trigonometric equations arising in real-world situations?

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- How can the unit circle explain symmetry with odd and even and periodicity of trigonometric functions?

Student will be able to:

- Use the radian measure of an angle as the length of the arc of the unit circle.
- Describe the unit circle and the basic trigonometric functions to shed light on the relationships between two quantities, interpreting the radian measure of an angle traverses counterclockwise around the unit circle.
- Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.
- Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
- Explain odd and even symmetry and periodicity of trigonometric functions using the unit circle.
- Write trigonometric functions that model periodic phenomena with specified amplitude, frequency, and midline.
- Restrict the domain of a trigonometric function on which the function is always increasing or always decreasing, allowing its inverse to be created.
- Uses inverse trigonometric functions to solve equations that arise in a modeling context, evaluating the solutions with or without technology and interpret the solutions in terms of a context.
- Prove the Pythagorean identities and use these identities to solve problems within a specific context.
- Use the sum and the difference of sine, cosine, and tangent to solve real world situations.

Evidence for Assessing Learning

Performance Tasks

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate solutions to problems from Purple Math lessons
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals
- Evaluate participation in group discussion and work

Building the Learning Plan

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Sample Classroom Activities and/or Lesson Plans:

- Provide an overview of trigonometric functions as shown below:

The Magic Rule of Trigonometry: $\sin^2 x + \cos^2 x = 1$

Note: The Magic Rule is derived using a Unit Circle with an inscribed right triangle and the Pythagorean Theorem.

($\sin x = \text{vertical distance}$)($\cos x = \text{horizontal distance}$)($\text{hypotenuse} = 1$)

Derivation #1 from the Magic Rule:

Given: $\sin^2 x + \cos^2 x = 1$

$$D(\sin^2 x) \quad \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

Simplify $1 + \cot^2 x = \csc^2 x$

or $1 = \csc^2 x - \cot^2 x$

or $\csc^2 x - \cot^2 x = 1$

Derivation #2 from the Magic Rule:

Given: $\sin^2 x + \cos^2 x = 1$

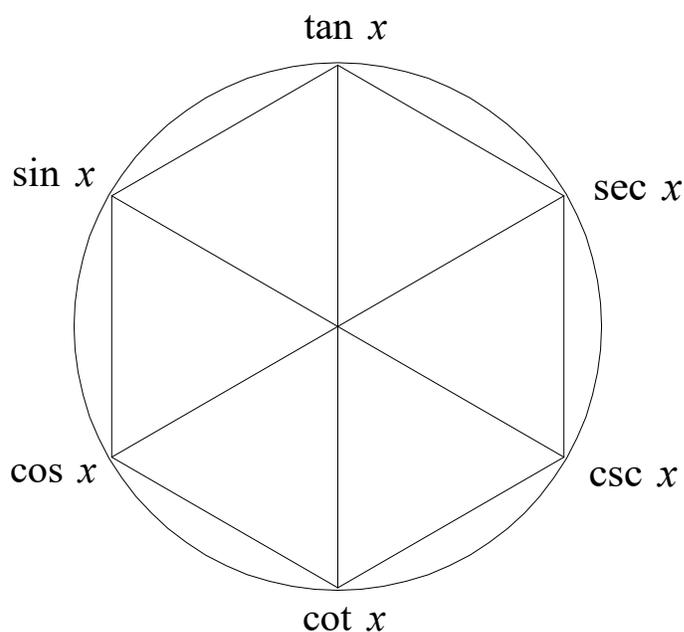
$$D(\cos^2 x) \quad \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

Simplify $\tan^2 x + 1 = \sec^2 x$

or $1 = \sec^2 x - \tan^2 x$

or $\sec^2 x - \tan^2 x = 1$

Trigonometric Identity Hexagon



Understanding the Trigonometric Identity Hexagon:

1. The two trigonometric functions at the ends of any diameter are reciprocals of one another.
2. Every trigonometric function is the product of the trigonometric functions on either side of it. For example: $\sin x = \tan x \cos x$ and $\cos x = \sin x \cot x$
3. Each trigonometric function is equal to either of its adjacent trigonometric functions divided by its adjacent trigonometric function. For example:

$$\tan x = \frac{\sin x}{\cos x} \quad \text{and, also} \quad \tan x = \frac{\sec x}{\csc x}$$

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4. The product of any three non-adjacent functions is always 1. For example: $\tan x \cos x \csc x = 1$ and $\sin x \cot x \sec x = 1$.

NOTES: This mnemonic won't work if the Trigonometric Identity Hexagon is not drawn with the trigonometric functions at the correct vertices. A key for reproducing the mnemonic is to remember sin, tan, and sec for the three vertices at the top of the hexagon. Their co-functions are immediately below them. A total of 26 trigonometric identities can be obtained from this mnemonic. Have the students work in groups to derive and prove the 26 trigonometric identities depicted in the Trigonometric Identity Hexagon.

- Present a mini-lesson on Trigonometric Functions from Purple Math
- Discuss content and model sample problems and solutions as whole class
- Students independently study the “The Unit Circle” lesson from Purple Math and record a summary of the content in their math journals

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Scientific calculator
- Notebook
- PurpleMath.com unit on “Trigonometric Functions and Their Graphs” (3 pages)
- PurpleMath.com unit on “The Unit Circle”

List of Technology Resources:

- Purple Math lesson on Trigonometric Functions - <http://www.purplemath.com/modules/triggrph.htm>
- Purple Math lesson on The Unit Circle - <http://www.purplemath.com/modules/unitcirc.htm>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

NRS Level 6 Math
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GEOMETRY (G)

Similarity, Right Triangles, and Trigonometry (SRT)

6.G.SRT.1 / 6.G.SRT.2 / 6.G.SRT.3 / 6.G.SRT.4 / 6.G.SRT.5 / 6.G.SRT.6

Essential Understandings:

- The ratios of the sides of right triangles are functions of the acute angles of the triangle.
- The sine of an acute angle in a right triangle is equal to the cosine of that angle's complement (and vice versa).
- The Pythagorean Theorem applies only to right triangles.
- Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of the triangle.
- Prove the Laws of Sine and Cosine for right triangle trigonometry in real-world situations.
- Prove the Laws of Sine and Cosine for non-right triangle trigonometry in real-world situations.

Essential Questions:

- How does similarity give rise to the trigonometric ratios?
- How do the trigonometric ratios of complementary angles relate to one another?
- How can the Pythagorean Theorem be used to solve problems involving triangles?
- How can the formula $A = \frac{1}{2}ab \sin(C)$ for the area of the triangle be derived and used?
- How can the Laws of Sine and Cosine for right triangle trigonometry in real-world situations be proved?
- How can the Laws of Sine and Cosine for non-right triangle trigonometry in real-world situations be proved?

Student will be able to:

- Use similarity to show that the side ratios in a right triangle are properties of the angles in the triangle.
- Use the definitions of trigonometric ratios for acute angle in a right triangle.
- Explain and use the relationship between sine and cosine of complementary angles.
- Apply and use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.
- Derive an area of a triangle formula using the sine trig function.
- Prove and use the Law of Sine and Law of Cosine on both right triangle trigonometry and non-right triangle trigonometry in real world situations.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed trigonometric functions worksheet
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals
- Monitor group work and discussion of Khan Academy practice section

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plan

- Preset a lesson on trigonometric functions as ratios of the sides of a right triangle using material from <https://www.khanacademy.org/math/geometry/hs-geo-trig>
- Review the six trigonometric functions
- Students work as a class to solve the five problems under “Practice: Solve for a Side in Right Triangles” on Khan Academy
- Present the following worksheet for independent practice, applying trigonometry ratios of:

$$\text{Sine} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tangent} = \frac{\text{opposite}}{\text{hypotenuse}}$$

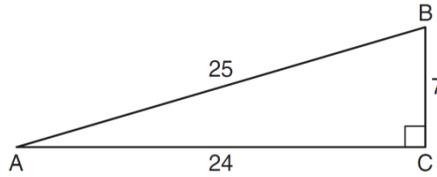
(The other three trig functions – cotangent, secant and cosecant are defined in terms of the first three. They are used less often, but they simplify some problems, as in the worksheet below)

$$\text{Cotangent} = 1/(\tan A) \quad \text{Secant} = 1/(\cos A)$$

$$\text{Cosecant} = 1/(\sin A)$$

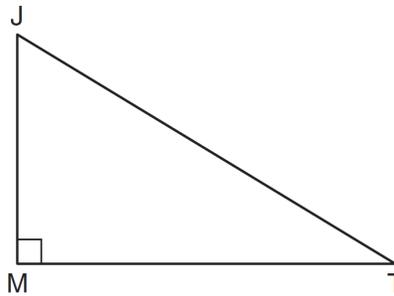
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1 Which ratio represents $\csc A$ in the diagram below?



- 1) $\frac{25}{24}$
- 2) $\frac{25}{7}$
- 3) $\frac{24}{7}$
- 4) $\frac{7}{24}$

2 In the diagram below of right triangle JTM , $JT = 12$, $JM = 6$, and $m\angle JMT = 90$.

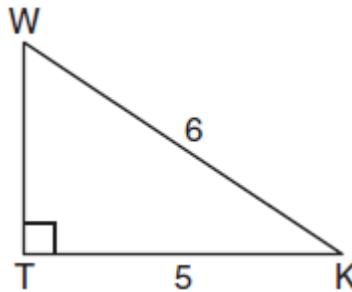


What is the value of $\cot J$?

- 1) $\frac{\sqrt{3}}{3}$
- 2) 2
- 3) $\sqrt{3}$
- 4) $\frac{2\sqrt{3}}{3}$

3 In the diagram below of right triangle KTW , $KW = 6$, $KT = 5$, and $m\angle KTW = 90$.

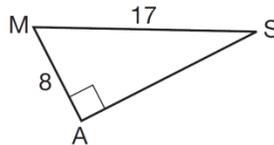
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What is the measure of $\angle K$, to the *nearest minute*?

- 1) $33^{\circ}33'$
- 2) $33^{\circ}34'$
- 3) $33^{\circ}55'$
- 4) $33^{\circ}56'$

- 4 In the right triangle shown below, what is the measure of angle S, to the *nearest minute*?



- 1) $28^{\circ}1'$
- 2) $28^{\circ}4'$
- 3) $61^{\circ}56'$
- 4) $61^{\circ}93'$

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- <https://www.khanacademy.org/math/geometry/hs-geo-trig#concept-intro>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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GEOMETRY (G)

Expressing Geometric Properties with Equations (GPE)

6.G.GPE.1 / 6.G.GPE.2 / 6.G.GPE.3 / 6.G.GPE.4

Essential Understandings:

- The equation of a circle can be found given a center and radius length.
- The equation of a parabola can be found given a focus and directrix.
- The equation of an ellipse and hyperbola can be found given the foci or the sum/difference of distances from the foci.
- Use coordinates to prove simple geometric theorems algebraically.

Essential Questions:

- How can the equation of a circle be found given a center and radius length?
- How can the equation of a parabola be found given a focus and directrix?
- How can the equation of an ellipse and hyperbola be found given the foci or the sum/difference of distances from the foci?
- Using coordinates, how can simple geometric theorems be proven algebraically?

Student will be able to:

- Write the equation of a circle given the circle's center and radius length.
- Write the equation of a parabola given the parabola's focus and directrix.
- Write the equation of an ellipse or hyperbola given the foci and/or the sum/difference of the distances of from the foci.
- Use the coordinates in a coordinate plane to prove simple geometric theorems algebraically.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate notes and practice problems from Khan Academy
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Evaluate level of participation in group discussion and work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on Conic Sections with material from Khan Academy PreCalc unit – Conic Sections

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- Use the videos and challenge/practice problems to work through the content as a whole group.
- Demonstrate how to derive an equation of a circle using the Pythagorean Theorem in the coordinate plane
- On an overhead projector or using smart room technology, model the following problem: A diagram of a circle with center at the origin and a radius with a length of 5 units is drawn in the coordinate plane. The points (5,0) (0,5), (-5,0), and (0,-5) are points on the circle. What other points are on the circle and what is the equation of the circle?
- Let $P(x,y)$ be any other point on the circle. From P, draw a vertical line segment to the x-axis. Let this be point Q. Then triangle OPQ is a right triangle with $OQ = x$, $PQ = y$ and $OP = 5$. We can then use the Pythagorean Theorem to write an equation of a circle: $OQ^2 + PQ^2 = OP^2$ or $x^2 + y^2 = 5^2$
- The points (3,4), (4,3), (-3,4), (-4,3), (-3,-4), -4,-3), (3,-4) and (4, -3) appear to be points on the circle and all make the equation $x^2 + y^2 = 5^2$ true. If we replace 5 by the length of any radius, r, the equation of a circle whose center is at the origin is: $x^2 + y^2 = r^2$
- Present a lesson on finding the equation of a parabola from Khan Academy (including the videos) and complete the practice problems as a whole class

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology – lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Overhead projector
- Graph paper and protractors
- Khan Academy unit on Conic Sections
- Khan Academy unit on “Focus and Directrix of a Parabola”

List of Technology Resources:

- Khan Academy “Conic Sections” - <https://www.khanacademy.org/math/precalculus/conics-precac>

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- Khan Academy “Focus and Directrix of a Parabola” - <https://www.khanacademy.org/math/algebra2/intro-to-conics-alg2/focus-and-directrix-of-a-parabola-alg2/v/focus-and-directrix-introduction>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G)

Geometric Measurement and Dimension (GMD)

6.G.GMD.1 / 6.G.GMD.2 / 6.G.GMD.3

Essential Understandings:

- Given an informal argument, explain the formulas for the circumference, area of a circle, volume of a cylinder, pyramid, and cone can be solved.
- Given an informal argument using Cavalieri’s principle, explain the formulas of a sphere and other solid figures can be solved.
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects.
- Identify three-dimensional objects generated by rotations of two-dimensional objects.

Essential Questions:

- How can an informal argument explain the formulas for the circumference, area of a circle, volume of a cylinder, pyramid, and cone?
- How can an informal argument using Cavalieri’s principle explain the formulas of a sphere and other solid figures?
- How can shapes of two-dimensional cross-sections of three-dimensional objects be identified?
- How can shapes of three-dimensional objects generated by rotations of two-dimensional objects be identified?

Student will be able to:

- Use an informal argument to explain the formulas for the circumference and area of a circle, the volume of a cylinder, a pyramid, and cone.
- Use Cavalieri’s principle to explain the formula of a sphere and other solid geometric figures.
- Identify and state the two-dimensional cross-section of a three-dimensional object.
- Identify the shapes of three-dimensional objects generated by rotations of two-dimensional objects.

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Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Written explanations for formulas (by group)
- Complete and accurate solutions to Khan Academy practice problems (by group or independently)
- Plato Learning Environment tests-monitor management system by objective and NRS level

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on solid geometry using material from Khan Academy unit on Solid Geometry
- Working in groups students discuss and write in words explanations for the formulas for circumference and area of a circle, and volume of a cylinder, pyramid, and cone
- Groups present their explanations and class votes for the best for each formula
- As a whole class, present a lesson on 2D vs. 3D objects using videos and material from Khan Academy
- Students complete the practice sections in small groups or independently

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology – lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Khan Academy Solid Geometry lesson and practice
- Khan Academy 2D vs. 3D Objects lesson and practice

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List of Technology Resources:

- Khan Academy – Solid Geometry
https://www.khanacademy.org/math/geometry/hs-geo-solids/hs-geo-solids-intro/e/solid_geometry
- Khan Academy – 2D vs. 3D Objects
<https://www.khanacademy.org/math/geometry/hs-geo-solids/hs-geo-2d-vs-3d/e/cross-sections-of-3d-shapes>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) Modeling with Geometry (MG) 6.G.MG.1 / 6.G.MG.2

Essential Understanding:

- Geometric objects may be used to model various physical phenomena.
- The concepts of density can be applied in modeling problems involving area or volume.

Essential Question:

- How can geometric figures be used to model physical phenomena or problem situations?
- What is density as it relates to area or volume?

Student will be able to:

- Use and solve various geometric objects to model physical phenomena or problem situations.
- Explain (in words) and describe (mathematically) how density relates to area and volume.

Evidence for Assessing Learning

Performance Tasks:

- Complete and accurate human figure based on geometric shapes
- Teacher-generated quizzes and tests on class vocabulary or activities
- Facilitated group work/ monitoring conversations and work with rubric

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Evaluate and monitor group and independent work
- Completed design project for farmer's market

Other Evidence:

- Math journals

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Option: open by showing a video from Inside Mathematics where a teacher helps student apply principles of quadrilaterals to the real-life problem of manufacturing kites. Use this as a prompt for discussion of other real-life application of math principles
- Present an art activity for using geometric shapes to represent a human body
- Begin by visualizing the human figure as a combination of simple geometric forms: ovals, cones, cylinders, circles, etc.
- Ask students to draw circles for the shoulders, elbows and wrists. Then connect them to form the arms. The elbows are next to the waist. The wrist is next to the hip joint. Draw the shoulders larger than the head. Draw the circles for the knees slightly lower than the tips of the fingers. To make the thighs, draw a line from the outside of the oval shaped knee. Draw a line from the inside of the hip to the inside of the knee. This ensures that the thigh is drawn wide enough at the top and narrows as it goes down.
- The entire leg, from the top of the hip to the sole of the foot, is half the length of the whole human figure (Demonstrate a model on the overhead)
- When students have completed their geometric figure, they place a thin piece of paper over their drawing and create a second one with more accurate proportions, omitting the circles for the joints. They can be creative and add clothes, shoes, and hair
- Explore design problems that exist in local communities, such as building a shed with maximum capacity in a small area or locating a dog park for three communities in a desirable area. Discuss challenges such as physical constraints and ways to minimize costs
- Propose a problem for groups to discuss: Maximize the number of vendors in a given space for a farmer’s market along a narrow path of a lot. Work with given constraints such as standard stall sizes, distance required between stalls, etc. Create a design and justify the work with geometric methods

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology – lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts

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- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Picture of local farmer's market and estimate of physical statistics such as area, perimeter, etc.
- Notebooks

List of Technology Resources:

- Inside Mathematics – Video of lesson applying principles of quadrilaterals to real-life problem <http://www.insidemathematics.org/common-core-resources/mathematical-practice-standards/standard-4-model-with-mathematics>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

STATISTICS AND PROBABILITY (SP)

Interpreting Categorical and Quantitative Data (ID)

6.S.ID.1 / 6.S.ID.2 / 6.S.ID.3 / 6.S.ID.4 / 6.S.ID.5 / 6.S.ID.6

Essential Understandings:

- Data in a two-way frequency table can be summarized using relative frequencies in the context of the data.
- A line of best fit can be generated for a set of data to model the relationship between two variables by hand or with technology.
- Correlation does not imply causation.
- Exponential functions, like linear, can be used to model real-life situations.
- Key features in graphs and tables shed light on relationships between two quantities.
- Differences exist between linear and exponential functions, and recognizing these differences will allow one to choose the appropriate model.
- Units, scale, data displays, and levels of accuracy represented in situations.
- Functions can be created to best fit data represented on a scatter plot.
- Computations and interpretations are used to decide if differences between parameters are significant.
- A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related.
- The properties of a situation or data set determine what type of function (e.g., linear, quadratic, exponential, polynomial, rational, or logarithmic) should be used to model it.

Essential Questions:

- What is the role of statistics in real-world situations?

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- From a scatterplot, how are two quantitative variables related?
- When is it appropriate to use causation or correlation?
- How can computations and interpretations help to determine which model is appropriate in a given situation?
- What are the key features of a linear, quadratic, or exponential function in a modeling situation?
- How can a situation best be modeled by a linear, quadratic, or exponential function?
- How are units, scale, data displays, and levels of accuracy selected to appropriately represent a situation?
- How can a function that best fits the data from a scatter plot be determined?
- How can a scatter plot that is created or interpreted from data fit a function?
- What are key characteristics to identify when choosing a function to model a given situation?

Student will be able to:

- Scatter plots may be used to represent data with two quantitative variables.
- Fit the best functions to the data in terms of the context of the situation, such as fitting line of best fit that suggests a linear association.
- Find the rate of change (the slope) and the intercept (a constant term) of a linear model in the context of the data.
- Use technology to find and interpret the correlation coefficient of a linear fit.
- Compare and contrast the characteristics between a correlation and causation.

Evidence for Assessing Learning

Performance Tasks:

- Facilitated group work/ monitoring conversations and work with rubric
- Complete and accurate group work demonstrating types of scatter plots
- Complete and accurate scatter plot from independent practice

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Review the identification of linear or nonlinear associations, as well as outliers and clusters in a scatter plot. Then show three different scatter plots on the overhead. In pairs, students identify each scatter plot as having positive, negative, or no association, whether the association is linear or nonlinear, and identify if the scatter plot has an outlier and/or cluster
- Before explaining to students how to write the equation of a line for the trend line, have students individually write down the steps of writing the equation of a line in their notes

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- After explaining how students can interpret the slope and the y-intercept, give the students a sample problem that includes a scenario that can be described by a linear equation, and have the students interpret the slope and the y-intercept given what each variable represents
- During the guided practice, students will create and analyze a scatter plot for a given set of data with guidance from the teacher
- During the independent practice, students will create and analyze a scatter plot for a given set of data.

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/by NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers
- Graph paper
- Measuring table
- M&Ms
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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STATISTICS AND PROBABILITY (SP)

Conditional Probability and the Rules of Probability (CP)

6.S.CP.1 / 6.C.SP.2 / 6.S.CP.3 / 6.S.CP.4 / 6.S.CP.5 / 6.S.CP.6 / 6.S.CP.7 / 6.S.CP.8 /
6.S.CP.9

Essential Understandings:

- Events can be described as a subset of a sample space.
- The probability of two events occurring together is the product of their probabilities, if and only if then the events are independent.
- The probability of two events can be conditional on each other and the interpretation of that probability.
- Two-way frequency tables can be used to decide if events are independent and to find conditional probabilities.
- Conditional probability and independence are applied to everyday situations.
- Conditional probability of A given B can be found and interpreted in context.
- The Addition or Multiplication Rule can be applied and the resulting probability can be interpreted in a context or in terms of a given model.
- Permutations and combinations can be used to compute probabilities of compound events in problem-solving situations.

Essential Questions:

- How can an event be described as a subset of outcomes using correct set notation?
- How are probabilities, including joint probabilities, of independent events calculated?
- How are probabilities of independent events compared to their joint probability?
- How does conditional probability apply to real-life events?
- How are two-way frequency tables used to model real-life data?
- How are conditional probabilities and independence interpreted in relation to a situation?
- What is the difference between compound and conditional probabilities?
- How is the probability of event (A or B) found?
- How can the Addition or Multiplication Rule be applied and the resulting probability be interpreted within a context or in terms of a given model?
- How can permutations and combinations be used to compute probabilities of compound events in problem-solving situations?

Student will be able to:

- Describe events as a subset of a sample space, using characteristics of categories of the outcomes, or as unions, intersections, or complements of other events.
- Use the product of two probabilities when the events in a given situation are independent events, using this characterization to determine if they are independent.

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- Create everyday situations that use conditional probability and independent events
- Find that the conditional probability of event A given event B and interpret this probability in the context given.
- Recognize, explain, and use the concepts of conditional probability and independence in everyday language and in everyday situations.
- Use the Addition or Multiplication Rule and interpret the appropriate rule in a context or in terms of a given model.
- Use permutations and combinations to compute probabilities of compound events in problem solving situation.
- Use and construct two-way frequency tables for independent events and find the conditional probability.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate problems in Steck Vaughn and New Readers Press text (if using as intervention or re-teaching option)
- Complete and accurate practice problems on Khan Academy (as group or independent work)
- Three complete and accurate worksheets from Explorations with Chance lesson (as group or independent work)
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on conditional probability, beginning with material from Khan Academy Unit “Randomized Algorithms”
- Have students work in groups to complete the challenge questions and problems
- During the next session, present the “Explorations with Chance” lesson from NCTM Illuminations website. Students will explore conditional probability and how to compute probability of a compound event by playing and analyzing three different games of chance
- Student complete “Is It Fair?” worksheet, “A Fair Hopper” Activity Sheet, and the “Happy Hopper” Activity Sheet, either in groups or independently

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- In their math journals students record their impressions of games of chance and the mathematical reasons for their opinions

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Review concepts of probability in a textbook context (New Readers Press and Steck-Vaughn materials – see below)
- Peer teaching through group work
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Handouts and lesson outline for Illuminations “Explorations with Chance” lesson
- Notebooks
- Coins
- Chips with faces of red-red, red-blue, red-white, and white-blue
- *Mathematical Reasoning: GED® Preparation for the 2014 Test*. (2013). Steck-Vaughn. (Unit 2 Lesson 4)
- *Scoreboost 2014 - Graphs, Data, and Probability*. (2013). New Readers Press. (pages 26-37)

List of Technology Resources:

- Khan Academy www.khanacademy.org
- Purple Math www.purplemath.com
- NCTM Illuminations – Lesson on Explorations with Chance
<http://illuminations.nctm.org/Lesson.aspx?id=1145>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways