

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 5**

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Acknowledgements

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NRS Level 5
Low Adult Secondary Education (Grade Levels 7 – 8)

NUMBER AND QUANTITY (N)
THE REAL NUMBER SYSTEM (RN)

5.N.RN.1 / 5.N.RN.2 / 5.N.RN.3 / 5.N.RN.4 / 5.N.RN.5 / 5.N.RN.6

Essential Understandings:

- Adding and multiplying two rational numbers results in a rational number.
- The result of adding a rational number and an irrational number is an irrational number.
- The result of multiplying a non-zero rational number to an irrational number is an irrational number.

Essential Questions:

- What is the relationship between properties of operations and types of numbers?
- Why are quantities represented in multiple ways?
- How can quantities be represented and what is the rationale for selecting a specific representation?
- What type of number results when adding or multiplying two rational numbers?
- What type of number results when adding a rational number to an irrational number?
- What type of number results when multiplying a non-zero rational number to an irrational number?

Student will be able to:

- Use prior knowledge of operations to add and subtract rational numbers and graph results on a line diagram.
- Apply and extend previous understanding of addition and subtraction of rational numbers by describing situations in which opposite quantities combine to make 0.
- Apply and extend previous understanding of addition and subtraction of rational numbers by showing understanding of $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative.
- Demonstrate that a number and its opposite are additive inverses, so their sum is 0, and interpret sums of rational numbers in real-world contexts.
- Apply and extend previous understanding of addition and subtraction of rational numbers by demonstrating the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- Demonstrate previous knowledge of addition and subtraction of rational numbers by applying properties of operations as strategies to add and subtract rational numbers.
- Use the properties of multiplication, division and fractions to multiply and divide rational numbers.
- Demonstrate the distributive property with signed numbers and apply to real world situations.

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- Apply division to integers with divisors other than zero, recognizing the quotient of integers is a rational number, and apply to real life situations.
- Use long division to convert a rational number to a decimal, recognizing the decimal form of rational numbers ends in 0 or repeats.
- Demonstrate understanding that every number has a decimal expansion of some kind and that rational numbers end in 0's or eventually repeat.
- Compare the size and value of irrational numbers by using rational number approximations, locate them on a number line diagram, and provide an approximate estimation of value.
- State or write (using words or examples) the difference between a rational and irrational number
- Add and multiply two rational numbers to obtain a rational number.
- Add a rational and irrational number to obtain an irrational number.
- Multiply a nonzero rational number and an irrational number to obtain an irrational number.
- Use the four operations with rational numbers to solve real-world and mathematical problems, including the ability to manipulate complex fractions.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes
- Textbook unit reviews and/or exercises as quizzes and chapter tests

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Extend examples of relationships between exponents and radicals through interactive technology
- Use practice worksheets to develop skills of rational exponent usage
- Students will work in groups, reasoning about general statements on rational and irrational numbers
- Students record thinking in their math journals: explaining and justifying their classification decisions
- Students should work independently to improve their individual solutions to the assessment task

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Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

(Core and supplemental)

- Mini white boards
- Calculators
- Possible Texts:
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *Common Core Achieve: Mathematics.* (2013). Contemporary/McGraw-Hill

List of Technology Resources:

- Kuta Software—<https://www.kutasoftware.com>
- Purple Math—www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

NUMBER AND QUANTITY (N)

QUANTITIES (Q)

5.N.Q.1 / 5.N.Q.2

Essential Understandings

- Rates, ratios, percentages and proportional relationships can be applied to multi-step ratio and percent problems along with other problem solving situations such as interest, tax, discount, etc.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Arithmetic and algebra can be used together, with the rules of conversion to transform units.
- Scales, graphs, and other data models can be interpreted.

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Essential Questions:

- When is it advantageous to represent relationships between quantities symbolically? numerically? graphically?
- How can the units used in a problem help determine a solution strategy?
- How can units, scale, data displays and levels of accuracy be chosen to appropriately represent a situation?

Student will be able to:

- Express the relationships between quantities symbolically, numerically, graphically and verbally when given a real-world situation or a mathematical context.
- Use appropriate units when obtaining an arithmetic or algebra solution to a real-world multi-step problem.
- Use and interpret appropriate units consistent with a given formula or multi-step problem (e.g., area will have square units and volume with cubed units).
- Choose and interpret scales and the origin on various types of graphs and/or data displays.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from texts
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities. (See class activities for possibilities)
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Make and interpret graphs, such as circle graph, bar graph, and line graph, based on real-life data (candy, student groups represented, classroom objects)
- Students will make frequency tables to go with their graphs
- Have students apply multiple discounts to a given item
- Have students calculate the rate of change in real-life situations. For example, calculate the change in attendance at the state fair from one year to another.

Learning Activities: (*differentiated instruction, re-teaching options*)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention

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- Peer teaching through group work
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Small packet of candies or lollipops
- Colored pencils or markers
- Poster board
- Rulers and glue
- Store advertisements
- *Mathematical Reasoning: Test Preparation for the 2014 GED TEST*. (2014). Steck- Vaughn.
- *Possible Texts:*
 - *Math 2: Algebraic Thinking, Data Analysis, and Probability* (2014). New Readers Press
 - *Data Analysis and Probability* (2016). McGraw Hill/College and Career Readiness) Practice Workbook.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways

ALGEBRA (A)

SEEING STRUCTURE IN EXPRESSIONS (SSE)

5.A.SSE.1

Essential Understandings:

- Identify and interpret the different parts of expressions that represent certain values contextually.
- Exponential expressions represent a quantity in terms of its context.

Essential Questions:

- What are the different ways to represent an exponential expression?
- How can an appropriate expression be created to model data or situations given within context?

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Student will be able to:

(what does mastery look like)

- Identify and interpret a term, its factors and its coefficient within any polynomial expression.
- Interpret expressions having grouping symbols by viewing one of more of the factors or parts as a single entity.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities (See class activities for possibilities)
- Assign outside projects as evaluated against a rubric
- Math journals-graded entries
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Students will work in groups to create mock financial records. Each member of the group will contribute ideas for keeping personal financial records
- Students will use the concept of exponential growth to model functions in real-life situations (i.e., the spread of colds and bacteria)

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Focus on a clear statement of an “exponential relationship”
- Have students graph points on a smart board or other interactive technology with teacher direction
- Have students make comparisons between linear and exponential graphs
- Have them write all findings in their math journal
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts

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- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers
- Legal pads, graphing paper
- Play money
- Markers
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw Hill/College and Career Readiness Practice Workbook
 - *EMPower Math: Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
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- iPathways

ALGEBRA (A)

CREATING EQUATIONS (CED)

5.A.CED.1 / 5.A.CED.2 / 5.A.CED.3 / 5.A.CED.4

Essential Understandings:

- Linear models can be created, used, and interpreted for real-life situations.
- Real world situations can be modeled by systems of linear equations.
- A system of equations can have no, one, or infinitely many solutions.
- Solutions of systems of inequalities are ordered pairs that satisfy all equations as well as inequalities that are often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Multiple methods may be used to solve a system of equation or inequalities.
- Functions can be created to best fit data represented on various models.
- Polynomial functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation.
- Systems can be solved graphically, algebraically or from a table.

Essential Questions:

- What real world situations can be modeled by a linear relationship?

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- How can technology help to determine whether a linear model is appropriate in a given situation?
- How can systems of linear equations or inequalities be used to model real world situations?
- How can the solution(s) of a system be represented and interpreted?
- What processes may be used to solve a system of equations or inequalities?
- How can a linear function be found that best fits data from various models?
- What are the different methods that can be used to find the solutions of a system of equations?
- When changes are made to an equation, what changes are made to the graph?
- What new information will be revealed if a formula is written in a different but equivalent form?
- How can the solution(s) of a system be represented and interpreted?

Student will be able to:

(what does mastery look like)

- Create and interpret various types of equations and inequalities in one variable, using them to solve problems.
- Create and interpret various types of equations in two or more variables, describing the relationship of the two quantities being represented and determining whether the solution(s) are viable or nonviable for the modeling context.
- Graph systems of either equations or inequalities on a coordinate axes, properly labeling and scaling the axes.
- Graph systems of equations or inequalities on a coordinate axes, properly labeling and scaling the axes.
- Rewrite formulas or literal equations to highlight a quantity of interest.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

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- Students survey different word problems that involve writing an equation with one variable. Then they write their own word problem that can be solved by a two-step equation.
- Students then experiment with different methods of solving systems of equations
- Demonstrate graphing equations and inequalities on a coordinate plane.
- Students experiment with graphing sample equations and inequalities in small groups

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring through group work
- Think out loud (demonstrate how to think about a problem)
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Straight edge
- Pencils
- Formula worksheet from GED® testing service:
<http://www.gedtestingservice.com/uploads/files/0756c16704434ff71e43c8117a5fa738.pdf>
- Possible Texts:
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

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- Purple Math www.purplemath.com
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ALGEBRA (A)
REASONING WITH EQUATIONS AND INEQUALITIES (REI)
5.A.REI.1 / 5.A.REI.2 / 5.A.REI.3 / 5.A.REI.4 / 5.A.REI.5

Essential Understandings:

- Algebraic concepts can be proven, and actions taken to arrive at a solution can be justified.
- The relationships between quantities can be explained or justified verbally in the exploration of real-world situations.
- The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous).
- Linear functions can be represented by a table, graph, verbal description or equation and that each representation can be transferred to another representation.
- Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).
- There are several ways to solve a quadratic equation (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the equation.
- The quadratic formula is derived from the process of completing the square.
- Complex numbers exist and can arise in the solutions of quadratic equations.
- A quadratic function that does not intersect the x -axis has complex zeros.
- The relationship between the factors of a quadratic and the x -intercepts of the graph of the quadratic.

Essential Questions:

- Why are procedures and properties necessary when manipulating numeric or algebraic expressions?
- How can the structure of an equation or an inequality help determine a solution strategy?
- What are complex numbers, and why do they exist?
- How can a quadratic equation be solved?
- How do the factors of a quadratic determine the x -intercepts of the graph and vice versa?
- How is the quadratic formula derived?

Student will be able to:

(what does mastery look like)

- Write a viable argument to justify each step used to find a solution in a simple equation.
- Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.
- Use the method of completing the square to derive the quadratic formula.

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- Solve quadratic equations in one variable for its zeros by either factoring, completing the square, using the quadratic formula, or using the square root property, recognizing that a solution(s) can be either real, irrational, or complex (written in a $\pm bi$ form).
- Use either the method of substitution or elimination to solve systems of equations.
- Find either an exact or approximate solution, focusing on systems of linear equations.
- Represent and solve equations and inequalities in two variables in the coordinate plane, explaining the set of all the solutions plotted.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Students work in groups of four. Each group has a bell. An equation is put on the Smart Board or other interactive technology. (This activity can be used for one-step, multiple steps, equations with two variables, or inequalities.) Groups ring the bell when they know the first step to solving the equation. The group to ring first and has the right answer earns a point for their team. They can converse with their group but have 30 seconds to answer. This continues until the problem is solved
- Have students answer concept questions in their math journals that involve systems of two linear equations in two variables. For example: 1) What can you say about the solution? 2) Explain what it means for a system to be independent. 3) Explain what it means for the system to be inconsistent
- Have students participate in self-check exercises working with a partner, followed by a discussion of all steps toward solutions

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on articulating a clear statement on “equation relationships”
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as:

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interactive websites/illustrations, Study Stack – vocabulary support

- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Rulers
- Pencils
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *EMPower Math: Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
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- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

FUNCTIONS (F)

INTERPRETING FUNCTIONS (IF)

5.F.IF.1

Essential Understandings:

- The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous).
- The zeros of each factor of a polynomial determine the x -intercepts of its graph.
- Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).

Essential Questions:

- How can a function and its notation be used, interpreted, and defined?
- How can you represent a function and what are the key features of each representation?
- What are the key features of a linear or quadratic function? Slope? Intercepts? Maxima? Minima?
- What type of linear, quadratic or exponential function is best to model a given situation?

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Student will be able to:

(what does mastery look like)

- Use paper-and-pencil to graph simple functions and use technology to graph more complicated functions, showing key features of the graph.
- Graph linear functions showing intercepts.
- Graph quadratic functions showing intercepts and either maxima or minima.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on graphing quadratic functions, focusing on the skill of creating tables and graphs.
- Introduce quadratics and the general form of the quadratic function and equation, demonstrating how to convert the quadratic equation into the quadratic function. Use a graph to give students a visual representation of what the key terms represent.
- Next show students how to graph two simple quadratic functions (i.e., $y=x^2$ and $y=3x^2$)
- Add additional examples in increasing difficulty and with negative coefficients for the quadratic terms.
- Present students with opportunities to participate in the lesson by encouraging them to come to the board to fill out the table of values or graph the quadratic function.

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on articulating a clear statement on “equation relationships”
- Teacher- directed class discussion on key features of the quadratic graph.
- Have students make comparisons between quadratic graphs and linear graphs
- Teaching through group work

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- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers and graph paper
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - *EMPower Math: Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
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- iPathways

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FUNCTIONS (F)
BUILDING FUNCTIONS (BF)
5.F.BF.1 / 5.F.BF.2

Essential Understandings:

- Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line.
- Arithmetic sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.

Essential Questions:

- What is an arithmetic sequence and how does it relate to linear functions?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- How can applied problems using quadratics be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (e.g., factoring to find the zeros, or completing the square to find the maximum or minimum)?

Student will be able to:

- Write a function and describe the relationship between the two quantities represented.
- Find an explicit expression or a recursive process and describe the steps for calculating an expression from a context.
- Create a linear or exponential function or an arithmetic or geometric sequence, given a graph, a verbal description or an input-output table, transferring easily between each of these representations to obtain a correct solution.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Math journals-graded entries
- Facilitated group work/ monitoring conversations and work with rubric

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present lesson on functions with a skill focus on creating tables, graphs, and equations
- Review key vocabulary: relationship, function, independent variable, dependent variables, linear relationship, linear graph, x-axis, y-axis, ordered pairs, explicit expression, recursive process
- On the overhead, show different types of functions that clearly describe a relationship between quantities
- Ask the questions: Where in real life can situations be found where the value of one variable depends on the value of another? How does changing the value of the independent variables affect the value of the dependent variables?
- Explain that the recursive process uses the previous term and the rate of change and that an explicit expression uses the rate of change and the initial value

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on articulating a clear statement on “equation relationships”
- Teacher-directed class discussion on key features of the quadratic graph.
- Have students make comparisons between quadratic graphs and linear graphs
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One on one intervention
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Transparency with different types of graphs
- Calculator
- Graph paper
- Rulers
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Intermediate Algebra* (2016) McGraw-Hill/College and Career Readiness Practice Workbook
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

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List of Technology Resources:

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- Purple Math www.purplemath.com
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- iPathways

GEOMETRY (G) CONGRUENCE (CO)

5.G.CO.1 / 5.G.CO.2 / 5.G.CO.3 / 5.G.CO.4 / 5.G.CO.5 / 5.G.CO.6 / 5.G.CO.7 /
5.G.CO.8 / 5.G.CO.9 / 5.G.CO.10 / 5.G.CO.11 / 5.G.CO.12 / 5.G.CO.13

Essential Understandings:

- When parallel lines are cut by a transversal, corresponding angles, alternate interior angles, alternate exterior angles, and vertical angles are congruent.
- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects represented in the coordinate plane.
- Any two geometric figures are congruent if there is a sequence of rigid motions (rotations, reflections, or translations) that carries one onto the other.
- A proof consists of a hypothesis and conclusion connected with a series of logical steps.
- The basic building blocks of geometric objects are formed from the undefined notions of point, line, distance along a line, and distance around a circular arc.
- Two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles of the triangles are congruent.
- It is possible to prove two triangles congruent without proving corresponding pairs of sides and corresponding pairs of angles of the triangle are congruent if certain subsets of these six congruence relationships are known to be true (e.g. SSS, SAS, ASA, but not SSA).
- Different observed relationships between lines, between angles, between triangles, and between parallelograms are provable using basic geometric building blocks and previously proven relationships between these building blocks and between other geometric objects.
- The geometric relationships that come from proving triangles congruent may be used to prove relationships between geometric objects.
- Geometric figures can be constructed using various tools, methods and relationships.

Essential Questions:

- In terms of rigid motions, when are two geometric figures congruent?
- What are the undefined building blocks of geometry and how are they used?
- What are possible conditions that are necessary to prove two triangles congruent?
- What are the roles of hypothesis and conclusion in a proof?
- What criteria are necessary in proving a theorem?

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- What is the significance of demonstrating the relationships between geometric figures through constructions?

Student will be able to:

- Write a definition based on the undefined notions of point, line, distance along a line and distance around a circular arc for an angle, a circle, a set of perpendicular or parallel lines, and line segments.
- Develop, describe, and draw a transformation or a sequence of transformations (translations, rotations, and reflections) of a given geometric figure as a function of input and output values or by using the coordinate plane.
- Develop, describe, and draw a transformation or a sequence of transformations (translations, rotations, and reflections) of a rectangle, parallelogram, trapezoid, or regular polygon as it can be carried onto itself by creating an input-output table of values or by using the coordinate plane.
- Compare and contrast a transformation (i.e., a translation versus a horizontal stretch) which preserves and does not preserve the distance and/or angular measure of a geometric figure in the coordinate plane.
- Use geometric descriptions of rigid motions to transform and predict the effect of a given rigid motion onto a given geometric figure.
- Use the definition of congruence in terms of rigid motions to decide if two geometric figures are congruent.
- Explain how the criteria for triangle congruence (i.e., ASA, SAS, SSS) follow the definition of congruence in terms of rigid motion.
- Prove theorems about lines, angles, triangles and parallelograms.
- Use various geometric tools to make formal geometric constructions, such as copying a segment or an angle, bisecting a segment and/or angle, constructing either perpendicular or parallel lines, and state the significance of demonstrating relationships between geometric figures through constructions.
- Construct an equilateral triangle, a square, or a regular hexagon inscribed in a circle with a compass and straightedge.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Give students visuals of congruent triangles accompanied with five postulates and theorems. Use two-column proofs to model all six components that prove congruency. The columns also will model the thought process in a logical and orderly way
- The second model will be a justification of the columns by citing postulates, theorems, or definitions
- Next, present students with a representation of two identical triangles. They must discuss reasons why the triangles are identical (congruent). Students list supporting postulates, theorems, and definitions
- Students work together in small groups, reasoning to draw conclusions and solve problems
- Give student groups clues, fill in blanks, and steps presented in order
- Students complete a worksheet with fifteen sample problems. Each problem is accompanied by a two-column fill-in, with postulates and theorems to choose from

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Teacher-directed class discussion on key features
- Have students make comparisons of different theorems
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Math manipulatives for drawing accurate geometric shapes
- Protractor
- Colored pencils
- Overhead projector
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Geometry and Measurement* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G)

SIMILARITY, RIGHT TRIANGLES AND TRIGONOMETRY (SRT)

5.G.SRT.1 / 5.G.SRT.2 / 5.G.SRT.3 / 5.G.SRT.4 / 5.G.SRT.5 / 5.G.SRT.6

Essential Understandings:

-
- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects represented in the coordinate plane.
- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged, and that the dilation of a line segment is longer or shorter in the ratio given by the scale factor of the dilation.
- Two geometric figures are similar if there is a sequence of similarity transformations (dilation along with rotations, reflections, or translations) that carries one onto the other.
- Two triangles are similar if and only if corresponding pairs of angles are congruent and corresponding pairs of sides are proportional.
- It is possible to prove two triangles similar by proving that two pairs of corresponding angles of the triangles are congruent.
- The Pythagorean Theorem can be used to find the missing side lengths in a coordinate plane and real-world situations.
- Different observed relationships between geometric objects are provable using basic geometric building blocks and previously proven relationships between these building blocks and between other geometric objects.
- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects.
- The properties of congruent and of similar triangles can be used to solve problems that either involve or can be modeled with triangles.

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

Essential Questions:

- What angle relationships are formed by a transversal intersecting with two parallel lines?
- What are the properties of dilations?
- In terms of similarity transformations, when are two geometric figures similar?
- What are the necessary conditions to know when two triangles are similar?
- What are the sufficient conditions to know that two triangles are similar?
- How can the Pythagorean Theorem be proven using the geometric relationships that come from proving triangles similar?
- How can the geometric relationships that come from proving triangles congruent or from proving triangles similar be applied in problems solving situations?
- How do indirect measurement strategies (using the Pythagorean Theorem) allow for the measurement of items in the real world such as playground structures, flagpoles, and buildings?

Student will be able to:

- Use proper terminology (e.g., reduction and/or enlargement) to describe how the properties of a dilation can be used on a geometric figure given either a center or a scale factor.
- Determine when two geometric figures are similar in terms of similarity transformations.
- Use the properties of similarity transformations to establish the Angle-Angle (AA) criteria for two similar triangles.
- Prove and apply theorems about triangles using congruence and similarity criteria.
- Develop facts about angle sums and exterior angles of triangles, angles created by parallel lines cut by transversals, and the angle-angle criterion.
- Find the distance between two points in a coordinate system by applying the Pythagorean Theorem.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Facilitated group work/ monitoring conversations and work with rubric

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson verifying experimentally the properties of dilations given by a center and scale factor
- Explain that dilations create parallel lines between ALL pre-image and image corresponding segments and lines
- The lesson should move slowly, starting with basic dilations with one ray. It is upon a ray that students will gain understanding of enlargement, reduction, and direction of dilation
- Definition: A dilation with the center O and a scale factor of K is a transformation that maps every point P in the plane P' so that the following properties are true:
 - 1) Angle measure (angles stay the same)
 - 2) Parallelism (parallel lines stay the same)
 - 3) Collinearity (points stay on the same lines)
 - 4) Midpoint (midpoints remain the same)
 - 5) Orientation (lettering order stays the same)
 - 6) Distance (distance is NOT preserved)
- After a dilation, the pre-image and image have the same shape but not the same size, and the dilation leaves a line unchanged.
- Use different worksheets with coordinate planes to give students opportunities to sketch dilation images and practice labeling

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Focus on dilation properties
- Focus on articulating a clear statement on similarity of geometric figures
- Teacher- directed class discussion on key features of Necessary conditions and sufficient conditions of triangle similarity
- Have students make comparisons between the three accepted methods of Proofs with Similar Triangles
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Protractor
- Overhead projector on Smart Board technology for viewing interactive dilations

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Geometry and Measurement* (2016). McGraw-Hill/College and Career Readiness Practice Workbook.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G) CIRCLES (C)

5.G.C.1 / 5.G.C.2 / 5.G.C.3 / 5.G.C.4 / 5.G.C.5

Essential Understandings:

- Rounded object volume can be calculated with specific formulas.
- Pi is necessary when calculating volume of rounded objects.
- Different relationships among inscribed angles, radii, and chords of a circle, and between the angles of a quadrilateral inscribed in a circle are provable using previously proven relationships between geometric objects.

Essential Questions:

- What are the different relationships among inscribed angles, radii, and chords of a circle, and of the angles of a quadrilateral inscribed in a circle?
- What is the relationship between the length of the arc of a circle, the central angle of the circle that intercepts the arc, and the radius of the circle?
- What is the area of a sector of a circle?

Student will be able to:

(what does mastery look like)

- Prove that all circles are similar.
- Identify central angles, inscribed angles, radii, and chords within a circle.
- Describe the different relationships amongst inscribed angles, radii, and the chords of a circle.
- Construct both inscribed and circumscribed circles of a triangle.
- Construct a quadrilateral inscribed in a circle and prove properties of angles for this quadrilateral.
- Construct a tangent line from a point outside a given circle to the circle.
- Determine that the length of an arc intercepted by an angle is proportional to the radius.
- Define the radian measure of an angle as a constant of proportionality.
- Find the area of a sector of circle.

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Provide groups with measurement tools, such as a protractor and rulers.
- Students draw a circle with a radius of their choice. Next they draw another circle that is a dilation of the first. Students can choose any size dilation they wish
- Each group should use the same measurement tool to measure and record the radii, diameter, and circumference of their circles
- Students sketch a graph on the coordinate plane of the diameters vs. the circumference of their circles
- Next, students find the slope of the resulting line using the graph
- Groups compare graphs and slopes and discuss the following questions:
1) What do you notice about the slopes? 2) Do you think this holds true for all circles? 3) What do we call this ratio?

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Pencils

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- Protractor
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G)

EXPRESSION GEOMETRIC PROPERTIES WITH EQUATIONS (GPE)

5.G.GPE.1 / 5.G.GPE.2 / 5.G.GPE.3

Essential Understandings:

- Geometric figures can be represented in the coordinate plane.
- That algebraic properties (including those related to the distance between points in the coordinate plane) may be used to prove geometric relationships.
- The distance formula may be used to determine measurements related to geometric objects represented in the coordinate plane (e.g., the perimeter or area of a polygon).
- The algebraic relationship between the slopes of parallel lines and the slopes of perpendicular lines.

Essential Questions:

- What is the relationship between the slopes of parallel lines and of perpendicular lines?
- Given a polygon represented in the coordinate plane, what is its perimeter and area?
- How can geometric relationships be proven through the application of algebraic properties to geometric figures represented in the coordinate plane?

Student will be able to:

(What does mastery look like?)

- Use the coordinate plane to describe and prove the relationship between the slopes of parallel and perpendicular lines to solve geometric problems,
- Find a point on a directed line segment that partitions the segment in a given ratio.
- Use the distance formula to determine the length of a segment or side of a polygon, using this information to find either the perimeter or area of this polygon.

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on the midpoint of a line segment
- Instruct students to find the midpoint of a line segment. Give each student a piece of 4-inch string licorice along with the following instructions.
 - Find the middle and lightly mark the half-way point without using a measurement tool.
 - Next, lay the licorice on graph paper finding the end point coordinates, and labeling them.
 - Connect the points to form a line segment.
 - Investigate ways to find the midpoint of the segment.
 - Write the midpoint as an ordered pair.
 - Describe how you found the midpoint of your line segment.
- Put students into groups of three or four. Give each group a worksheet with two pairs of coordinates per one graph. They will graph the pairs of points, connect the points to form a line segment, and create a formula for the midpoint. They must find the midpoint using one method they have selected by their group. If that method does not work, groups experiment to discover another that will work.
- Teacher should monitor groups and ask leading questions: 1) How did you find the midpoint, given two end points? 2) Can you write a formula for finding the midpoint using variables and equal signs? 3) Can you find the slope of a line between the two end points? 4) What is the equation of a line in slope intercept form?

Learning Activities:

- Teacher- directed class discussion on key features.
- Have students make comparisons between the slope of a line and the midpoint of a line.
- Peer teaching through group work
- Interactive technology- as assigned by instructor to support instruction-such as: interactive websites/illustrations, Study Stack – vocabulary support, etc.)- see below
- One-to-one intervention

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Straight edge
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test.* (2014). Steck- Vaughn.
 - *Common Core Achieve.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G)

GEOMETRIC MEASUREMENT AND DIMENSION (GMD)

5.G.GMD.1 / 5.G.GMD.2 / 5.G.GMD.3

Essential Understanding:

- The formulas for circumference, area, surface area, and volume of two- and three-dimensional geometric figures can be seen as linear and other functions of the radius.

Essential Question:

- How can familiar formulas for two-and three-dimensional geometric figures be viewed as a function and/or model?
- How is the volume and/or surface area of various three-dimensional geometric objects determined?

Student will be able to:

(What does mastery look like)

- Demonstrate knowledge of formulas for volume of cylinders, cones, and spheres by applying them to and finding solutions for real-world and mathematical problems.
- Find the circumference and area of a circle using algebra.
- Find the volume for cylinders, pyramids, cones and spheres.

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- Describe (in words) how the formulas from various two-dimensional figures can be incorporated into the formulas for the volume of three-dimensional figures (e.g., the area of a circle is the base of the cylinder times the height of the cylinder).

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Chapter reviews/tests from textbooks:
- Teacher-generated quizzes and tests on class exercises, vocabulary, or activities
- Assign outside projects as evaluated against a rubric
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on finding the volume of a cylinder
- Review key vocabulary: Area, Base, Circle, Circumference, Pi, Radius, Volume
- Draw a circle on a Smart Board, label the diameter, radius, and circumference and give the diameter a numerical value
- Students use a calculator to find the area of the circle
- Draw a cylinder and write the following notes on the board:
 - A cylinder has parallel and equal sized circle as bases. To find the volume of a cylinder, multiply the area of a base by the height of the cylinder: $V=Bh$, V =volume, B = area of a base, h = height
 - Since the base of a cylinder is always a circle, substitute the formula for the area of a circle into the formula for the volume. $V=Pi(r^2)(h)$
- Next, perform a demonstration to determine the volume of a cylinder
- Math manipulates are one 2 inch diameter cylinder flower vase and one 4 inch diameter cylinder flower vase (both are 14 inches tall). Pour a cup of water in the first. Ask students how much of the water in the first vase will fill the second. Pour the same water from the first vase into the second vase. Students will see that it is not half full. Show them mathematically why it is $\frac{1}{4}$.
Vase one: $V= Pi(1^2)(12)$ Vase two: $V= Pi(2^2)(12)$. The ratio is 1:4

Learning Activities:

- Teacher- directed class discussion on key features of the lesson.
- Have students make comparison between cone volumes and cylinder volumes
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Interactive websites/illustrations, Study Stack – vocabulary support

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- 2-inch diameter flower vase
- 4-inch diameter flower vase
- 4 cups of water
- Calculators
- Circle and cylinder worksheets
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software <https://www.kutasoftware.com>
- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

GEOMETRY (G)

MODELING WITH GEOMETRY (MG)

5.G.MG.1

Essential Understandings:

- Different geometric objects can be used to model the same or various physical phenomena and the object chosen to model the phenomena will be dependent upon how the model is to be used.
- The concept of density and how it may be applied in modeling problems involving area or volume.

Essential Questions:

- How can geometric properties and relationships be applied to solve problems that are modeled by geometric objects?
- What is density as it relates to area or volume?

Student will be able to:

- Use geometric properties and relationships in real-world applications that model a geometric object (e.g., a school track or field may be created by a rectangle with two half-circles placed at the opposite ends of the rectangle or a silo can be created by a cylinder with half of a sphere atop it).

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- Explain (in words) and describe (mathematically) how density relates to area and volume.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated quizzes and tests on class vocabulary or activities
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present an art activity for using geometric shapes to represent a human body
- Begin by visualizing the human figure as a combination of simple geometric forms: ovals, cones, cylinders, circles, etc.
- Ask students to draw circles for the shoulders, elbows and wrists. Then connect them to form the arms. The elbows are next to the waist. The wrist is next to the hip joint. Draw the shoulders larger than the head. Draw the circles for the knees slightly lower than the tips of the fingers. To make the thighs, draw a line from the outside of the oval shaped knee. Draw a line from the inside of the hip to the inside of the knee. This ensures that the thigh is drawn wide enough at the top and narrows as it goes down.
- The entire leg, from the top of the hip to the sole of the foot, is half the length of the whole human figure (Demonstrate a model on the overhead)
- When students have completed their geometric figure, they place a thin piece of paper over their drawing and create a second one with more accurate proportions, omitting the circles for the joints. They can be creative and add clothes, shoes, and hair

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

List of Instructional Materials:

- Math manipulatives such as geometric solids
- Drawing paper
- Sketching pencils or charcoals
- Measurement tools

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

STATISTICS AND PROBABILITY (S)

INTERPRETING CATEGORICAL and QUANTITATIVE DATA (ID)

5.S.ID.1 / 5.S.ID.2 / 5.S.ID.3 / 5.S.ID.4 / 5.S.ID.5 / 5.S.ID.6 / 5.S.ID.7 / 5.S.ID.8

Essential Understandings:

- Data can be represented and interpreted in a variety of formats (dot plots, histograms, and box plots).
- Extreme data points (outliers) can skew interpretations of a set of data.
- Synthesizing information from multiple sets of data results in evidence-based interpretation.
- Statistics provide information about a population (data set) by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.
- Center and spread of a data set may be compared in multiple ways.
- Random sampling tends to produce representative samples and support valid inferences.
- Two data distributions can be compared using visual and numerical representations based upon measures of center and measures of variability to draw conclusions.
- Data in a two-way frequency table can be summarized using relative frequencies in the context of the data.
- A line of best fit can be generated for a set of data to model the relationship between two variables by hand or with technology.
- A line of best fit aims to minimize the vertical distances between the data points and the points on the line and may be used to make predictions within the proximity of the data.
- Making predictions for values within or near the data set is more reliable than for values far beyond the data set.
- Correlation does not imply causation.
- Exponential functions, like linear, can be used to model real-life situations.

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- Key features in graphs and tables shed light on relationships between two quantities.
- Differences between linear and exponential functions, thus allowing them to use the appropriate model.
- Units, scale, data displays, and levels of accuracy represented in situations.
- Functions can be created to best fit data represented on a scatter plot.
- Computations and interpretations are used to decide if differences between parameters are significant.
- A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related.
- The mean and standard deviation of a data set is used to fit a normal distribution.
- Statistics is a process of making inferences.
- Different data collection methods are appropriate for different situations and randomization relates to each.
- Functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation.
- Exponential functions can be determined from data and used to represent many real-life situations (population growth, compound interest, depreciation, etc.).
- The properties of a situation or data set determine what type of function (e.g., linear, quadratic, exponential, polynomial, rational, or logarithmic) should be used to model it.

Essential Questions:

- What is the role of statistics in real-world situations?
- When is it appropriate to question the results from a model compared to real-life situations?
- Which data collection method is best used for a specific context?
- How does randomization relate to a data collection method?
- How can two data distributions be compared?
- How can statistics be used to gain information about a sample population?
- How can a random sample be used to draw inferences of a larger population?
- How is a population mean estimated from data from a sample survey?
- When is the difference between parameters significant?
- From a scatterplot, how are two quantitative variables related?
- How is a data set fit to a normal curve?
- How do various representations of data lead to different interpretations of the data?
- When and how can extreme data points impact interpretation of data?
- Why are multiple sets of data used?
- How are center and spread of data sets described and compared?
- How is a data set represented in a two-way frequency table summarized?
- When is it appropriate to use causation or correlation?

NRS Level 5

Low Adult Secondary Education (Grade Levels 7 – 8)

- How can computations and interpretations help to determine which model is appropriate in a given situation?
- What are the key features of a linear, quadratic, or exponential function in a modeling situation?
- How can a situation best be modeled by a linear, quadratic, or exponential function?
- How are units, scale, data displays, and levels of accuracy selected to appropriately represent a situation?
- How can a function that best fits the data from a scatter plot be determined?
- How can a scatter plot that is created or interpreted from data fit a function?
- What are key characteristics to identify when choosing a function to model a given situation?

Student will be able to:

- Use sampling to draw inferences about a population to show that statistics can be used to gain information by examining a sample of the population, and explain that random sampling tends to produce representative samples and support valid inferences.
- Represent and interpret data using a variety of formats (e.g., dot plots, histograms, box plots), ensuring that units, scales, data displays, and levels of accuracy represent the situation appropriately.
- Use statistics appropriately to shape data distributions and to compare the measures of central tendency (the median and mean) and spread (the interquartile range and standard deviation) of two or more different data sets.
- Interpret differences in shape, center and spread in terms of a context, accounting for possible extreme data points (outliers)
- Use the mean and standard deviation of a data set fit to a normal distribution and state whether the data set for which such a procedure is *or* is not appropriate.
- Summarize categorical data for two categories in two-way frequency table.
- Interpret relative frequencies (including joint, marginal, and conditional relative frequencies) in the context of the data, recognizing possible associations and trends in the data.
- Draw comparative inferences about two populations by informally assessing the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- Scatter plots may be used to represent data with two quantitative variables.
- Fit the best functions to the data in terms of the context of the situation, such as fitting line of best fit that suggests a linear association.
- Use technology to obtain the line of best fit and will aim to minimize the vertical distances between data points and the points on the line.
- Find the rate of change (the slope) and the intercept (a constant term) of a linear model in the context of the data.
- Use technology to find and interpret the correlation coefficient of a linear fit.

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- Compare and contrast the characteristics between a correlation and causation.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Facilitated group work/ monitoring conversations and work with rubric
- Complete and accurate group work demonstrating types of scatter plots
- Complete and accurate scatter plot from independent practice

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Introduce the concept of the scatter plot. Ask the students to recall the four different types of slope (positive, negative, zero, and undefined) and have students come up to the board to show what those types of slope look like
- Before explaining the different types of associations, have the students draw three blank scatter plots (no points) on a sheet of paper and label them Positive Association, Negative Association, and No Association. They will use M&Ms to show what they think the different types of associations look like. Then ask them to add two more blank scatter plots and label them Linear Association and Nonlinear Association. The students will again use M&Ms to show what those types of associations look like
- Review the identification of linear or nonlinear associations, as well as outliers and clusters in a scatter plot. Then show three different scatter plots on the overhead. In pairs, students identify each scatter plot as having positive, negative, or no association, whether the association is linear or nonlinear, and identify if the scatter plot has an outlier and/or cluster
- Before explaining to students how to write the equation of a line for the trend line, have students individually write down the steps of writing the equation of a line in their notes
- After explaining how students can interpret the slope and the y-intercept, give the students a sample problem that includes a scenario that can be described by a linear equation, and have the students interpret the slope and the y-intercept given what each variable represents
- During the guided practice, students will create and analyze a scatter plot for a given set of data with guidance from the teacher
- During the independent practice, students will create and analyze a scatter plot for a given set of data

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Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Peer tutoring
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Rulers
- Graph paper
- Measuring table
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

MAKING INFERENCES and JUSTIFYING CONCLUSIONS (IC)

5.S.IC.1 / 5.S.IC.2 / 5.S.IC.3 / 5.S.IC.4 / 5.S.IC.5 / 5.S.IC.6

Essential Understandings:

- Statistics can be a tool for making inferences about population versus sample parameters.
- Results from a model may or may not be consistent with real-life situations of the process.
- Different data collection methods are appropriate for different situations and randomization relates to each.
- Data from a sample survey are used to estimate a population mean.
- Real-life situations are used to decide if differences between parameters are significant.
- A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related.
- The mean and standard deviation of a data set are used to fit a normal distribution.
- Every day decisions are made based on data collection and interpretation.

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Essential Questions:

- How can statistics be used to understand parameters of a population versus the sample population?
- When is it appropriate to question the results from a model compared to real-life situations?
- Which data collection method is best used for a specific context?
- How does randomization relate to a data collection method?
- How is a population mean estimated from data from a sample survey?
- From a scatterplot, how are two quantitative variables related?
- How is a data set fit to a normal curve?
- How can reports or publications be evaluated based on the data presented?

Student will be able to:

(what does mastery look like)

- Uses statistics to understand the parameters of a population versus a sample population, explaining how data from a sample survey can be used to estimate a population mean.
- Use a simulation to decide if a specified model is consistent with results from a given data-generating process.
- Determine the purposes of and differences among sample surveys, experiments, and observational studies, explaining how randomization relates to the data in each.
- Use data from a randomized experiment to compare two treatments, using simulations to decide if differences between the parameters are significant.
- Explain how reports or publications can be evaluated based upon the data being presented.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate plot of given data
- Complete and accurate recording of slope, intercept, and equation of their line
- Facilitated group work/ monitoring conversations and work with rubric

Building the Learning Plan

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Low Adult Secondary Education (Grade Levels 7 – 8)

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on evaluating reports based on data
- Students choose two variables that they think may have a relationship (e.g., height vs. weight)
- Students collect data either from classmates, classroom, or online
- Students create a coordinate plane with an x and y axis labeled with two chosen variables, a number scale, and title
- Students plot the given data
- Students then draw a best-fit line for their data. If the data has no correlation, they can draw a line to continue with the forthcoming parts of the activity
- After students have drawn the line, they gather the slope and intercept from their line and find the equation of the line
- A teacher-directed discussion can summarize the positive, negative, and no correlation findings, encouraging students to compare their findings with those of classmates

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutor
- Focus on articulating a clear statement on “equation relationships”
- Peer teaching through group work

List of Instructional Materials:

- Graph paper
- Rulers
- Data collection sheets
- Suggested ideas on data that can be collected

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

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STATISTICS AND PROBABILITY (SP)
USING PROBABILITY TO MAKE DECISIONS (MD)

5.S.MD.1 / 5.S.MD.2 / 5.S.MD.3 / 5.S.MD.4 / 5.S.MD.5 / 5.S.MD.6 / 5.S.MD.7

Essential Understanding:

- Written descriptions, tables, graphs, and equations are useful in representing and investigating decision-making relationships in everyday life and work.

Essential Question:

- How are written descriptions, tables, graphs, and equations used in representing and investigating decision-making relationships in everyday life and work?

Student will be able to:

(what does mastery look like)

- Write descriptions with defined variables of interest that are useful in representing and investigating decision-making relationships in everyday life or work and support these descriptions with mathematical data.
- Graph a corresponding probability distribution using the same graphical displays as for data distributions.
- Find the expected value of a random variable, interpreting it as the mean of the probability distribution.
- Create a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated, finding an expected value.
- Create a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically, finding an expected value.
- Find, evaluate, and compare strategies on the basis of an expected payoff for a game of chance.
- Use probabilities to make fair decisions by drawing lots or using a random number generator.
- Use different probability concepts to analyze decisions and strategies within the context of a situation.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Facilitated group work/ monitoring conversations and work with rubric
- Completed “Practice: Expected Value” section on Khan Academy

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on random variables using selected material from <https://www.khanacademy.org/math/statistics-probability/random-variables-stats-library>
- Use the videos and problems from the “Discrete and continuous random variables and probability models” and “Expected value” sections of Khan Academy to discuss and model the content
- Students complete the “Practice: Expected Value” section and attempt the five questions, using as few hints as necessary

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- One-to-one tutor
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Additional practice with concepts and procedures in different contexts

List of Instructional Materials:

- Graph paper
- Khan Academy <https://www.khanacademy.org/math/statistics-probability/random-variables-stats-library>
- *Common Core Achieve*. (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Purple Math www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways