Illinois ABE/ASE Mathematics Content Standards

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Foreword

What Are Content Standards?

Content standards describe what learners should know and be able to do in a specific content area. The Illinois ABE/ASE Content Standards broadly define what learners who are studying reading, writing, and math should know and be able to do as a result of ABE/ASE instruction at a particular level. Content standards also help teachers ensure their students have the skills and knowledge they need to be successful by providing clear goals for student learning.

The Illinois ABE/ASE Content Standards should be used as a basis for curriculum design and may also be used to assist programs and teachers with selecting or designing appropriate instructional materials, instructional techniques, and ongoing assessment strategies. Standards do not tell teachers how to teach, but they do help teachers figure out the knowledge and skills their students should have so that teachers can build the best lessons and environments for their classrooms.

Why Are the Illinois ABE/ASE Content Standards Necessary?

The Illinois ABE/ASE Content Standards serve multiple purposes. They:

- Provide a common language for ABE/ASE levels among programs;
- Assist programs with ABE/ASE curriculum development;
- Provide guidance for new ABE/ASE instructors; and
- Ensure quality instruction through professional development.

Provide a Common Language for ABE/ASE Levels
ABE/ASE classes are very different across Illinois programs. The Illinois ABE/ASE Content Standards provide a description of what students should learn at each National Reporting System (NRS) level so that adult education practitioners have a common language when discussing ABE/ASE levels. Having a common language among levels and programs will help ABE/ASE learners who move from level to level within the same program or who move from one ABE/ASE program to another.

We need standards to ensure that all students, no matter where they live in the state of Illinois, are prepared for success in postsecondary education and/or the workforce. The Illinois ABE/ASE Content Standards will help ensure that our students are receiving a consistent education from program to program across the state. These standards will provide a greater opportunity to share experiences and best practices within and across the state that will improve our ability to best serve the needs of our students.

Assist Programs with ABE/ASE Curriculum Development

The Illinois ABE/ASE Content Standards should serve as the basis for a program's curriculum development process. For programs with an existing curriculum, that curriculum should be aligned to the standards. For programs without a curriculum, the standards provide an excellent framework and starting point for the curriculum development process.

Provide Guidance for New ABE/ASE Instructors

The Illinois ABE/ASE Content Standards provide guidance for new instructors who may have limited training in teaching adults enrolled in adult basic classes. The standards serve as a basis for what they should teach and include in their lesson plans.

Ensure Quality Instruction through Professional Development

In order to implement the Illinois ABE/ASE Content Standards, program staff (administrators and instructors) will participate in professional development on implementation of the standards. These professional development sessions will address curriculum design, instructional materials, instructional techniques, and ongoing assessment strategies related to the standards. They will also provide an excellent opportunity for new and experienced ABE/ASE instructors to develop and refine their teaching skills.

Why Were the Illinois ABE/ASE Content Standards Revised?

The GED® 21st Century Initiative will introduce a new assessment to our students in January 2014. The GED® 21st Century Initiative is committed to helping more adults become career- and college-ready by transforming the GED® test into a comprehensive program. By building a more robust assessment, complete with preparation tools and transitions to college and careers, GED® Testing Service and ACE hope to increase the number of adults who can enter and succeed in college and the workforce. The new assessment will be closely aligned with the Common Core State Standards and will be administered through computer-based testing (CBT), although paper-based testing (PBT) will still be available under certain circumstances or as an accommodation.

The Common Core State Standards Initiative is a state-led effort to establish a shared set of clear educational standards for English language arts and mathematics that states can voluntarily adopt. The standards have been informed by the best available evidence and the highest state standards across the country and globe and designed by a diverse group of teachers, experts, parents, and school administrators. These standards are designed to ensure that students graduating from high school are prepared to go to college or enter the workforce. The standards are benchmarked to international standards to guarantee that our students are competitive in the emerging global marketplace. The Illinois State Board of Education adopted the Common Core State Standards in June 2010.

In April 2013, the Office of Vocational and Adult Education (OVAE) released the highly-anticipated report, College and Career Readiness (CCR) Standards for Adult Education[1]. The report was the result of a nine-month process that examined the Common Core State Standards from the perspective of adult education. It was funded to provide a set of manageable yet significant CCR standards that reflect broad agreement among subject matter experts in adult education about what is desirable for adult students to know to be prepared for the rigors of postsecondary education and training.

How Were the Illinois ABE/ASE Content Standards Revised?

The original Illinois ABE/ASE Content Standards and Benchmarks (April 2011) were the result of several federal and state initiatives that addressed the need for content standards in adult education programs. During September 2011 a statewide application process was opened to adult education teachers, administrators, transition coordinators, etc., in order to participate in the ABE/ASE Content Standard Project. Selected applicants were assigned to either the math, reading, or writing team and began work in November 2011. The task for each team was to align the original Illinois ABE/ASE Content Standards and Benchmarks (April 2011) with the Common Core State Standards, College Readiness Standards, Career Clusters Essential Knowledge and Skills, Evidence Based Reading Instruction, the International Society for Technology in Education's National Educational Technology Standards for Students, and other standards to ensure student success in post-secondary education and/or employment.

The teams spent over six months reviewing, aligning, and editing the ABE/ASE content standards. A draft was submitted to the Illinois Community College Board (ICCB) in

[1] College and Career Readiness (CCR) for Adult Education, 2013, www.lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf.

February 2012. Select content area experts reviewed the draft in April 2012, and the standards went through an open comment period in May 2012.

After the release of the College and Career Readiness (CCR) Standards for Adult Education in April 2013, the Illinois ABE/ASE Content Standards that were published in June 2012 were reexamined. Because the content standards were already aligned with the Common Core State Standards, very few revisions were necessary. Additions have been made to the Reference column to highlight the CCR Standards that have been identified by OCTAE (Office of Career, Technical, and Adult Education – formerly known as OVAE). Furthermore, a gap analysis of the Illinois ABE/ASE Content Standards (June 2012) with the CCR Standards for Adult Education was completed by federal representatives in April 2014. The gap analysis examined the degree of alignment between the Illinois ABE/ASE Content Standards and key advance in the CCR Standards for Adult Education. The gap analysis concluded that “the standards – as they are presently composed – have many strengths, particularly the organization and structure of the standards document.”

In August 2019, the National Reporting System (NRS) released updated Educational Functioning Level Descriptors as part of the Technical Assistance Guide for Performance Accountability. The descriptors use the CCR Standards as a foundation and address the most critical mathematics concepts for adult learners. These descriptors provided a shift in mathematics content within the different NRS levels not previously noted. In 2020, a committee of math specialists began work accommodating the shifts in the standards and revising the example column of the standard framework. This work culminated in the most recent revision of the Illinois ABE/ASE Mathematics Standards.

Design of the Illinois ABE/ASE Content Standards

Adult education programs nationwide use the NRS educational functioning levels to provide information to the federal government about student progress. This uniform implementation makes it possible to compare data across programs. The Illinois ABE/ASE content standards conform to the NRS structure for consistency and accountability. There are six NRS educational functioning levels from beginning literacy and adult basic education through adult secondary education. (Note: The titles of the levels and the grade level equivalencies are different in Mathematics than in Language Arts.)The six levels each have titles and are identified by grade equivalency:

	NRS Mathematics Educational Functioning Levels	*Grade Level Equivalency*
1	Beginning ABE Literacy	0-1
2	Beginning Basic Education	2-3
3	Low Intermediate Basic Education	4-5
4	Middle Intermediate Basic Education	6-7
5	High Intermediate Education	7-8
6	Adult Secondary Education	9-12

	NRS Language Arts Educational Functioning Levels	*Grade Level Equivalency*
1	Beginning ABE Literacy	0-1
2	Beginning Basic Education	2-3
3	Low Intermediate Basic Education	4-5
4	High Intermediate Basic Education	6-8
5	Low Adult Secondary Education	9-10
6	High Adult Secondary Education	11-12

Content standards for reading, writing, speaking, listening, and math skills are included in this document. The essential knowledge and skills statements from the Career Clusters have also been incorporated into the content standards. The use of technology has been infused in this document as well. We would also like to reinforce that because these standards have been aligned to the Common Core State Standards, we are ensuring that our students are college and career ready. These standards are by no means meant to limit a teacher’s creativity. Certainly, some of the best teaching is done across the curriculum including some or all of the subject areas.

Assessment

Ongoing assessment of the Illinois ABE/ASE Content Standards should be a part of every lesson. Learners can demonstrate their mastery of a particular standard through ongoing assessment strategies such as demonstrations, project-based learning, presentations, simulation, out-of-class activities, and other nontraditional assessment strategies. Ongoing assessment is an integral part of instruction in standards-based education.

Introduction to the Mathematics Standards[2]

The Illinois ABE/ASE Math Common Core State Standards provide a consistent, clear understanding of what adult learners are expected to learn, so adult educators know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our students need for success in college and careers.

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These standards are a substantial answer to that challenge.

It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these standards aim for clarity and specificity.

William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

> *Articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline,* ***but also the key ideas*** *that determine how knowledge is organized and generated within that discipline. This implies that “to be coherent,” a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).*

These standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

[2] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing “sequenced obstacles and challenges for students…absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.

Adult learners need to envision mathematics as part of their daily language, a tool, and an art form with which they can communicate ideas, solve problems, and explore the world around them. When adult educators integrate technology and the Illinois State Career Clusters in their delivery of instruction, learners will be able to connect to, use, and express mathematical ideas in multiple ways in real life situations. Learners will also make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Through this process, they will develop mathematical reasoning skills and become adept at evaluating their own thinking.

Adult learners will have an understanding of how numbers are used and represented. They will be able to estimate and use basic operations to both solve everyday problems and confront more involved calculations in algebraic and statistical settings. They will be able to read, write, visualize, and talk about ways in which mathematical problems can be solved in both theoretical and practical situations. They will be able to communicate relationships in geometric and statistical settings through drawings and graphs.

Finally, as adult learners transition from level to level, they will demonstrate an understanding and use a variety of resources and tools to communicate their thinking from situation to situation and achieve additional academic knowledge and skills required to pursue a full range of career and postsecondary education opportunities.

Understanding Mathematics[3]

These standards define what ABE/ASE adult learners should understand and be able to do in their study of mathematics. Asking a student to understand something means asking an adult educator to assess whether the adult learner has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards are level specific but do not define the intervention methods or materials necessary to support adult learners who are well below or well above level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners (ELL) and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary to succeed and transition to college and careers in their post-school lives. The standards should be read as allowing for the widest possible range of adult learners to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs.

ELL students are capable of participating in mathematical discussions as they learn English. Mathematics instruction for ELL students should draw on multiple resources and modes available in classrooms---such as objects, drawings, inscriptions, and gestures---as well as mathematical experiences outside of the classroom.

Promoting a culture of high expectations for all students is a fundamental goal of the Illinois ABE/ASE State Standards. In order to participate with success in the general curriculum, both ELL and students with disabilities should be provided the necessary supports, accommodations, and services where applicable.

No set of level-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of adult learners in any given classroom.

[3] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

However, the standards do provide clear signposts along the way to the goal of college and career readiness for all students.

The standards begin here with eight Standards for Mathematical Practice.

Illinois ABE/ASE Standards for Mathematical Practice

The Illinois ABE/ASE Standards for Mathematical Practice describe a variety of expertise that mathematics adult educators at all levels should seek to develop in their adult students[4]. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). See Appendix B for a quick reference guide to the math practices.

1. Make sense of problems and persevere in solving them.

Mathematically proficient adult learners start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Higher level students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient adult learners can explain correspondences between equations, verbal descriptions, tables, and graphs, or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Lower level students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems

[4] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient adult learners make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize (to abstract a given situation, represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents); and the ability to contextualize (to pause as needed during the manipulation process in order to probe into the referents for the symbols involved). Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient adult learners understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient adult learners are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Lower level adult learners can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until higher levels. Later, students learn to determine domains to which an argument applies. Students at all levels can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient adult learners can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In lower levels, this might be as simple as writing an addition equation to describe a situation. In intermediate levels, adult learners might apply proportional reasoning to plan a school event or analyze a problem in the community. By the upper levels, an adult learner might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient adult learners who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient adult learners consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient adult students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient advanced level adult learners analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient adult learners try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the lower levels, adult learners give carefully formulated explanations to each other. By the time they reach the advanced levels, they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient adult learners look closely to discern a pattern or structure. Lower level students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, higher level students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient adult learners notice if calculations are repeated and look both for general methods and for shortcuts. Intermediate level students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, intermediate level students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient adult students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content[5]

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the literacy to intermediate and adult secondary levels. All adult educators, instructional leaders, designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Adult learners who lack understanding of a topic may rely on procedures too heavily. Without a flexible base

[5] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents an adult learner from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Guiding Principles for Mathematics Programs[6]

The following six Guiding Principles are philosophical statements that underlie the Standards for Mathematical Practice, Standards for Mathematical Content, and other resources in this ABE/ASE curriculum framework. They should guide the construction and evaluation of mathematics programs in adult education throughout the state of Illinois. The Standards for Mathematical Practice are interwoven throughout the Guiding Principles.

<u>Guiding Principle 1: Learning</u>
Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Adult learners need to understand mathematics deeply and use it effectively. The Standards for Mathematical Practice describe ways in which students increasingly engage with the subject matter as they grow in mathematical maturity and expertise through the six NRS levels.

To achieve mathematical understanding, adult learners should have a balance of mathematical procedures and conceptual understanding. Adult learners should be actively engaged in doing meaningful mathematics, discussing mathematical ideas, and applying mathematics in interesting, thought-provoking situations. Student

[6] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

understanding is further developed through ongoing reflection about cognitively demanding and worthwhile tasks.

Tasks should be designed to challenge ***every*** adult learner in multiple ways. Short- and long-term investigations that connect procedures and skills with conceptual understanding are integral components of an effective mathematics program. Activities should build upon curiosity and prior knowledge, and enable students to solve progressively deeper, broader, and more sophisticated problems. (See Standard for Mathematical Practice 1: *Make sense of problems and persevere in solving them.*) Mathematical tasks reflecting sound and significant mathematics should generate active classroom talk, promote the development of conjectures, and lead to an understanding of the necessity for mathematical reasoning. (See Standard for Mathematical Practice 2: *Reason abstractly and quantitatively.*)

Guiding Principle 2: Teaching

An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

The sequence of topics and performances should be based on what is known about how adult learners' mathematical knowledge, skill, and understanding develop over time. What and how students are taught should reflect not only the topics within mathematics but also the key ideas that determine how knowledge is organized and generated within mathematics. (See Standard for Mathematical Practice 7: *Look for and make use of structure.*) Adult learners should be given the opportunity to apply their learning and to show their mathematical thinking and understanding. This requires adult educators who have a deep knowledge of mathematics as a discipline.

Mathematical problem solving is the hallmark of an effective mathematics program. Skill in mathematical problem solving requires practice with a variety of mathematical problems as well as a firm grasp of mathematical techniques and their underlying principles. Armed with this deeper knowledge, the adult learner can then use mathematics in a flexible way to attack various problems and devise different ways of solving any particular problem. (See Standard for Mathematical Practice 8: *Look for and express regularity in repeated reasoning.*) Mathematical problem solving calls for reflective thinking, persistence, learning from the ideas of others, and going back over one's own work with a critical eye. Adult learners should be able to construct viable arguments and critique the reasoning of others. They should analyze situations and justify their conclusions, communicate their conclusions to others, and respond to the arguments of others. (See Standard for Mathematical Practice 3: *Construct viable arguments and critique the reasoning of others.*) Adult learners at all levels should be able to listen or read the arguments of others, decide whether they make sense, and ask questions to clarify or improve the arguments.

Mathematical problem solving provides students with experiences to develop other mathematical practices. Success in solving mathematical problems helps to create an abiding interest in mathematics. Adult students learn to model with mathematics and to apply the mathematics that they know to solve problems arising in everyday life, society, and the workplace. (See Standard for Mathematical Practice 4: *Model with mathematics.*)

For a mathematics program to be effective, it must also be taught by knowledgeable adult educators. According to Liping Ma, "The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics."[7] A landmark study in 1996 found that students with initially comparable academic achievement levels had vastly different academic outcomes when teachers' knowledge of the subject matter differed.[8] The message from the research is clear: having knowledgeable teachers really does matter; teacher expertise in a subject drives student achievement. "Improving teachers' content subject matter knowledge and improving students' mathematics education are thus interwoven and interdependent processes that must occur simultaneously."[9]

Guiding Principle 3: Technology

An essential tool that should be used strategically in mathematics education.

Technology enhances the mathematics curriculum in many ways. Tools such as measuring instruments, manipulatives (such as base ten blocks and fraction pieces), scientific and graphing calculators, and computers with appropriate software, if properly used, contribute to a rich learning environment for developing and applying mathematical concepts. However, appropriate use of calculators is essential; calculators should not be used as a replacement for basic understanding and skills. Adult education students that are in (levels 1-3) should learn how to perform the basic arithmetic operations independent of the use of a calculator.[10] Although the use of a graphing calculator can help adult learners in (levels 4-6) to visualize properties of functions and their graphs, graphing calculators should be used to enhance their understanding and skills rather than replace them.

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[7] Ma, Liping, *Knowing and Teaching Elementary Mathematics*, NYC: Taylor and Francis Routledge, 2010.

[8] Milken, Lowell, *A Matter of Quality: A Strategy for Answering the High Caliber of America's Teachers*, Santa Monica, California: Milken Family Foundation, 1999.

[9] Ma, p. 147.

[10] National Center for Education Statistics, *Pursuing Excellence: A Study of U.S. Fourth-Grade Mathematics and Science Achievement in International Context*. Accessed June 2000.

Adult educators and adult learners should consider the available tools when presenting or solving a problem. Adult learners should be familiar with tools appropriate for their level to be able to make sound decisions about which of these tools would be helpful. (See Standard for Mathematical Practice 5: *Use appropriate tools strategically.)*

Technology enables adult learners to communicate ideas within the classroom or to search for information in external databases such as the Internet, an important supplement to a program's internal library resources. Technology can be especially helpful in assisting students with special needs in all classrooms, at home, and in the community.

Technology changes the mathematics to be learned, as well as when and how it is learned. For example, currently available technology provides a dynamic approach to such mathematical concepts as functions, rates of change, geometry, and averages that was not possible in the past. Some mathematics becomes more important because technology requires it, some becomes less important because technology replaces it, and some becomes possible because technology allows it.

Guiding Principle 4: Equity

All students should have a high quality mathematics program that prepares them for college and a career.

All Illinois adult education program students should have a high quality mathematics program that meets the goals and expectations of these standards and addresses students' individual interests and talents. The standards provide clear signposts along the way to the goal of college and career readiness for all students. The standards provide for a broad range of students, from those requiring tutorial support to those with talent in mathematics. To promote achievement of these standards, adult educators should encourage classroom talk, reflection, use of multiple problem solving strategies, and a positive disposition toward mathematics. They should have high expectations for all students. At every level adult educators should act on the belief that every student should learn challenging mathematics. Adult educators and student services, where applicable, should advise adult learners about why it is important to take advanced courses in mathematics and how this will prepare them for success in college and the workplace.

All adult learners should have the benefit of quality instructional materials, good libraries, and adequate technology. All students must have the opportunity to learn and meet the same high standards. In order to meet the needs of the greatest range of students, mathematics programs should provide the necessary intervention and support for those students who are below or above specific level expectations. Practice and enrichment should extend beyond the classroom (i.e., tutoring sessions and mathematics computerized activities that promote learning).

Because mathematics is the cornerstone of many disciplines, a comprehensive curriculum should include applications to everyday life and modeling activities that demonstrate the connections among disciplines. (See Standard for Mathematical Practice 4: *Model with mathematics.*)

An important part of preparing students for college and careers is to ensure that they have the necessary mathematics and problem-solving skills to make sound financial decisions that they face in the world every day, including setting up a bank account; understanding student loans; reading credit and debit statements; selecting the best buy when shopping; and choosing the most cost effective cell phone plan based on monthly usage.

Guiding Principle 5: Literacy Across the Content Areas

An effective mathematics program builds upon and develops students' literacy skills and knowledge.

Reading, writing, and communication skills are necessary elements of learning and engaging in mathematics, as well as in other content areas. Supporting the development of students' literacy skills will allow them to deepen their understanding of mathematics concepts and help them to determine the meanings of symbols, key terms, and mathematics phrases, as well as to develop reasoning skills that apply across the disciplines. In reading, adult educators should consistently support students' ability to gain and deepen understanding of concepts from written material by helping them acquire comprehension skills and strategies, as well as specialized vocabulary and symbols. Mathematics programs should make use of a variety of text materials and formats, including textbooks, math journals, contextual math problems, and data presented in a variety of media.

In writing, adult educators should consistently support students' ability to reason and achieve deeper understanding of concepts, and to express their understanding in a focused, precise, and convincing manner. Mathematics classrooms should incorporate a variety of written assignments ranging from math journals to formal written proofs. In speaking and listening, adult educators should provide students with opportunities for mathematical discourse using precise language to convey ideas, communicate solutions, and support arguments. (See Standard for Mathematical Practice 6: *Attend to precision.*)

<u>Guiding Principle 6: Assessment</u>

Effective *assessment of student learning in mathematics should take many forms to inform instruction and learning.*

A comprehensive assessment program is an integral component of an instructional program. It provides adult learners with frequent feedback on their performance, adult educators with diagnostic tools for gauging students' depth of understanding of mathematical concepts and skills, and administrators with a means for measuring adult learners' achievement.

Assessments take a variety of forms, require varying amounts of time, and address different aspects of student learning. Having students "think aloud" or talk through their solutions to problems permits identification of gaps in knowledge and errors in reasoning. By observing adult learners as they work, adult educators can gain insight into students' abilities to apply appropriate mathematical concepts and skills, make conjectures, and draw conclusions. Homework, mathematics journals, portfolios, oral performances, and group projects offer additional means for capturing students' thinking, knowledge of mathematics, facility with the language of mathematics, and ability to communicate what they know to others. Tests and quizzes assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving; they can also pinpoint areas in need of more practice or teaching. Taken together, the results of these different forms of assessment provide rich profiles of adult learners' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.

Assessment should also be a major component of the learning process. As students help identify goals for lessons or investigations, they gain greater awareness of what they need to learn and how they will demonstrate that learning. Engaging adult learners in this kind of goal-setting can help them reflect on their own work, understand the standards to which they are held accountable, and take ownership of their learning.

How to Read the Six NRS Level Standards[11]

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Standards define what students should understand and be able to do.

Domain

Measurement and Data			
Measure lengths indirectly and by iterating length units			
1.MD.4	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	CC.1.MD.1	Understanding a child's growth chart Comparing coffee cup sizes
1.MD.5 **MWOTL**	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	CC.1.MD.2 ESS01.03.04 CCR.MD.A	Measuring the length and width of a photo

Cluster

Standard

The number of each standard is listed in the left hand column of each NRS level. The first number refers to the NRS level of the standard. The next two letters indicate the standard area by code; and the final number indicates the number of the skill within each domain. For example, "1.MD.4" labels NRS Level 1 Math standard #4 in the Measurement and Data domain.

Below the standard code a MWOTL notation may or may not be present. MWOTL stands for Major Work of the Level. Standards with this notation should be the focus of instructional time within the NRS level. Standards without the MWOTL present, support the MWOTL standards and are also essential to building the foundation of math for future levels of math knowledge.

[11] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

These standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given level, it does not necessarily mean that topic A must be taught before topic B. An adult educator might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, an adult educator might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What adult students can learn at any particular level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know A should next come to learn B." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity, therefore, level placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers, and mathematicians. One promise of the Illinois ABE/ASE state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across the state of Illinois adult education programs, and educators should make every effort to meet the needs of all students based on their current understanding.

The reference column lists the standard to which the ABE/ASE standard is aligned. Reference sources include: Common Core State Standard (CC)[12], States' Career Cluster Initiative Essential Knowledge and Skill Statements (ESS)[13], and the International Society for Technology in Education's National Educational Technology Standards for Students (NETS•S)[14] and the College and Career Readiness (CCR) Standards for Adult Education[15].

The fourth and final column within the framework contains examples of math tasks, activities and/or example questions that can be used to help instructors define the standard or demonstrate the standard within a classroom. The examples listed are not exhaustive but rather a quick reference for instructors. Each ICCB-funded program has an approved ABE/ASE Model Curriculum. This model curriculum should be referenced for more detailed lesson and activity recommendations.

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[12] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

[13] Career Cluster™ Knowledge and Skills, 2008, www.careertech.org.

[14] National Educational Technology Standards for Students, Second Edition, ©2007, ISTE® (International Society for Technology in Education), www.iste.org.

[15] College and Career Readiness (CCR) for Adult Education, 2013, www.lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf.

The Standards for Mathematical Content

NRS Levels 1- 6

NRS Level 1 Overview

Counting and Cardinality / Numeracy (CC)

- Know number names and the count sequence.
- Count to tell the number of objects
- Compare numbers

Operations and Algebraic Thinking (OA)

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
- Represent and solve problems involving addition and subtraction
- Understand and apply properties and the relationship between addition and subtraction
- Add and subtract within 20
- Work with addition and subtraction equations

Number and Operations in Base Ten (NBT)

- Extend the counting sequence
- Work with numbers 11-19 and tens to gain foundations for place value
- Use place value understanding and properties of operations to add and subtract

Measurement and Data (MD)

- Describe and compare measurable attributes
- Classify objects and count the number of objects in each category
- Measure lengths indirectly and by iterating length units
- Tell and write time
- Represent and interpret data

Geometry (G)

- Identify and describe shapes
- Analyze, compare, create, and compose shapes
- Reason with shapes and their attributes

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 1 – Beginning ABE Literacy
(Grade Levels 0 – 1)

At the Beginning ABE Literacy Level, instructional time should focus on four critical areas[16]:

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
2. Developing understanding of whole number relationships and place value, including grouping in tens and ones;
3. Developing understanding of linear measurement and measuring lengths as iterating length units; and
4. Describing, reasoning about attributes of, composing and decomposing geometric shapes.

(1) Adult learners develop strategies for adding and subtracting whole numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, students build their understanding of the relationship between addition and subtraction.

(2) Adult learners develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Adult learners develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

(4) Adult learners compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they

[16] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical understanding and expertise.

NRS Level 1 – Beginning ABE Literacy
(Grade Levels 0-1)

Standard Number	MATH STANDARD	Reference	Example task, problem, or activity
COUNTING AND CARDINALITY / NUMERACY (CC)			
Know number names and the count sequence.			
1.CC.1	Count to 100 by ones and by tens.	CC.K.CC.1 ESS01.03.01	Counting dimes, $10 bills Counting years 1yr, 10yr (decade), 100 years (century)
1.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	CC.K.CC.2 ESS01.03.01	Counting page numbers read
1.CC.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	CC.K.CC.3 ESS01.03.01	Counting items in a group Counting pencils in a pack
Count to tell the number of objects.			
1.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	CC.K.CC.4	Counting coins to pay for a purchase Telling which address falls in a given block, knowing the first number on the block Counting items on a store shelf

1.CC.5	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	CC.K.CC.5 ESS01.03.01	Counting the number of loose coins in a pile Counting the number of candies in a jar Counting the number of chairs/desks in a room
Compare numbers.			
1.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching and counting strategies). Include groups with up to ten objects.	CC.K.CC.6 ESS01.03.03	Separating loose coins into like piles and counting the number in each Separating items on a shelf by type or kind and comparing those groups
1.CC.7	Compare two numbers between 1 and 10 presented as written numerals.	CC.K.CC.7 ESS01.03.03	Comparing price tags on two items
OPERATIONS AND ALGEBRAIC THINKING (OA)			
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.			
1.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem.	CC.K.OA.1 ESS01.03.02	Figuring the number of hours of work or sleep by using fingers to count
1.OA.2	Solve addition and subtraction word problems, and add and subtract within 10 (e.g., by using objects or drawings to represent the problem).	CC.K.OA.2 ESS01.03.02	Counting money and making change Counting the total number of desks in a classroom

1.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., by using objects or drawings), and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	CC.K.OA.3 ESS01.03.02	Using manipulatives to establish number relationships
1.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number (e.g., by using objects or drawings), and record the answer with a drawing or equation.	CC.K.OA.4 ESS01.03.02	Using manipulatives to establish number relationships
1.OA.5	Fluently add and subtract within 5.	CC.K.OA.5 ESS01.03.02	Counting money and making change Checking out or returning library books
Represent and solve problems involving addition and subtraction.			
1.OA.6	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).	CC.1.OA.1 ESS01.03.02	Working out the shortfall in numbers (e.g., eggs for a recipe, plants to fill a display tray, cups to serve visitors)
1.OA.7	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem).	CC.1.OA.2 ESS01.03.02 CCR.OA.A	Finding the total price of 3 items ordered from a menu

Understand and apply properties and the relationship between addition and subtraction.			
1.OA.8 MWOTL	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (associative property of addition). Students need not use formal terms for these properties.	CC.1.OA.3 ESS01.03.02 CCR.OA.A	Placing same number of cookies on different shaped trays
1.OA.9 MWOTL	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.	CC.1.OA.4 ESS01.03.02 CCR.OA.A	Figuring change to receive from a $10 bill
Add and subtract within 20.			
1.OA.10 MWOTL	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	CC.1.OA.5 ESS01.03.02 CCR.OA.A	Watching a digital clock count down the time
1.OA.11 MWOTL	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and reading equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	CC.1.OA.6 ESS01.03.02 CCR.OA.A	Paying a $12 charge with a $10 bill and two $1 dollar bills

Work with addition and subtraction equations.			
1.OA.12	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.	CC.1.OA.7 ESS01.03.02 CCR.OA.A	Counting money and making change
1.OA.13 MWOTL	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?.	CC.1.OA.8 ESS01.03.03 CCR.OA.A	Test taking when seeking employment Helping children with homework
NUMBER AND OPERATIONS IN BASE TEN (NBT)			
Extend the counting sequence.			
1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	CC.1.NBT.1 ESS01.03.01	Counting page numbers read at one time, starting from first page read Counting inventory in a store
Work with numbers 11-19 and tens to gain a foundation and understand place value.			
1.NBT.2 MWOTL	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	CC.K.NBT.1 CC.1.NBT.2 ESS01.03.01 CCR.NBT.A	Using mental math to check that correct change was received
1.NBT.3 MWOTL	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	CC.1.NBT.3 ESS01.03.03 CCR.NBT.A	Telling which address falls in a block, knowing the house number

Use place value understanding and properties of operations to add and subtract.			
1.NBT.4 MWOTL	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	CC.1.NBT.4 ESS01.03.02 CCR.NBT.A	Calculating the production shortfall from a daily target Verifying deposits in a checking account Adding the total number of points on a test Saving money for a $100 purchase
1.NBT.5 MWOTL	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	CC.1.NBT.5 ESS01.03.02 CCR.NBT.A	Adding using mental math
1.NBT.6 MWOTL	Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	CC.1.NBT.6 ESS01.03.02 CCR.NBT.A	Counting money and making change Finding the temperature change between seasons
MEASUREMENT AND DATA (MD)			
Describe and compare measurable attributes.			
1.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	CC.K.MD.1	Describing a rectangular photo or frame Describing the shape of a book
1.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two people and describe one person as taller/shorter.*	CC.K.MD.2 ESS01.03.03 ESS01.03.04	Describing seasons, daylight savings time, or tides

Classify objects and count the number of objects in each category.			
1.MD.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.	CC.K.MD.3 ESS01.03.01	Sorting laundry or bottles for the recycling facility Organizing a store shelf by item
Measure lengths indirectly and by iterating length units.			
1.MD.4	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	CC.1.MD.1	Understanding a child's growth chart Comparing coffee cup sizes
1.MD.5 MWOTL	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	CC.1.MD.2 ESS01.03.04 CCR.MD.A	Measuring the length and width of a photo
Tell and write time.			
1.MD.6	Tell and write time in hours and half-hours using analog and digital clocks.	CC.1.MD.3	Reading a bus schedule that uses a.m. and p.m. Reading a class schedule Using a time clock for work
Represent and interpret data.			
1.MD.7	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	CC.1.MD.4 ESS01.03.06 CCR.MD.A	Understanding a child's growth chart Understanding an income based on education chart Creating a visual representation of favorite foods

GEOMETRY (G)			
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).			
1.G.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	CC.K.G.1	Recognizing the shape and meaning of a triangular yield sign and other shapes in buildings and everyday structures
1.G.2	Correctly name shapes regardless of their orientations or overall size.	CC.K.G.2	Identifying things by shape
1.G.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three- dimensional ("solid").	CC.K.G.3	Building a three dimensional model from flat material
Analyze, compare, create, and compose shapes.			
1.G.4 MWOTL	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	CC.K.G.4 CCR.G.A	Using a "Rules of the Road" book to describe shapes of road signs
1.G.5	Model shapes in the world by building shapes from components and drawing shapes.	CC.K.G.5	Using shapes to replicate campus map and roadways
1.G.6	Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"	CC.K.G.6	Building furniture that comes in pieces
Reason with shapes and their attributes.			
1.G.7	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	CC.1.G.1	Checking for quality control
1.G.8 MWOTL	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism."	CC.1.G.2 CCR.G.A	Building furniture that comes in pieces Arranging different objects/items on a shelf

1.G.9	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	CC.1.G.3	Cutting pizza, cake, and brownies

NRS Level 2 Overview

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving addition and subtraction
- Add and subtract within 20
- Work with equal groups of objects to gain foundations for multiplication
- Represent and solve problems involving multiplication and division
- Understand properties of multiplication and the relationship between multiplication and division
- Multiply and divide within 100
- Solve problems involving the four operations and identify and explain patterns in arithmetic

Number and Operations in Base Ten (NBT)

- Understand place value
- Use place value understanding and properties of operations to add and subtract and to perform multi-digit arithmetic

Number and Operations – Fractions (NF)

- Develop understanding of fractions as numbers

Measurement and Data (MD)

- Measure and estimate lengths in standard units
- Relate addition and subtraction to length
- Work with time and money
- Represent and interpret data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition
- Geometric measurement: recognize perimeter as an attribute of plan figures an distinguish between linear and area measures

Geometry (G)

- Reason with shapes and their attributes

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 2 – Beginning Basic Education
(Grade Levels 2- 3)

At the Beginning Basic Education Level, instructional time should focus on six critical areas[17]:

1. Extending understanding of base-ten notation;
2. Building fluency with addition and subtraction;
3. Developing understanding of multiplication and division and strategies for multiplication and division within 100;
4. Developing understanding of fractions, especially unit fractions (fractions with numerator 1);
5. Using standard units of measure and developing understanding of the structure of rectangular arrays and of area; and
6. Describing and analyzing one- and two-dimensional shapes.

(1) Adult learners extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Adult learners use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Adult learners develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division

[17] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(4) Adult learners develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(5) Adult learners recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. When building, students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(6) Adult learners describe, analyze, and compare properties of one- and two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. They investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical understanding and expertise.

NRS Level 2 – Beginning Basic Education
(Grade Levels 2-3)

Standard Number	MATH STANDARD	Reference	Example task, problem, or activity
OPERATIONS AND ALGEBRAIC THINKING (OA)			
Represent and solve problems involving addition and subtraction.			
2.OA.1 MWOTL	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions *(*e.g., by using drawings and equations with a symbol for the unknown number to represent the problem*).*	CC.2.OA.1 ESS01.03.02 ESS01.03.05 CCR.OA.B	Carrying out a stock inventory Checking grocery receipt against purchases
Add and subtract within 20.			
2.OA.2 MWOTL	Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one-digit numbers.	CC.2.OA.2 ESS01.03.02 CCR.OA.B	Estimating the bill at a restaurant
Work with equal groups of objects to gain foundations for multiplication.			
2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by pairing objects or counting them by 2s); write an equation to express an even number as a sum of two equal addends.	CC.2.OA.3 ESS01.03.02 ESS01.03.03	Telling which side of a street a house will be on from its number
2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	CC.2.OA.4 ESS01.03.02	Finding out how many chairs are needed for a meeting
Represent and solve problems involving multiplication and division.			
2.OA.5 MWOTL	Interpret products of whole numbers (e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each). For example, describe a context in which a total number of objects can be expressed as 5 x 7.	CC.3.OA.1 ESS01.03.02 CCR.OA.B	Determining how many pieces of pie you will have with multiple pies Determining the number of students in a class when placed in groups

2.OA.6 MWOTL	Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).	CC.3.OA.2 ESS01.03.02 CCR.OA.B	Splitting a restaurant bill (check) into equal parts for 2, 3, 4, 5 or more people
2.OA.7 MWOTL	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).	CC.3.OA.3 ESS01.03.02 ESS01.03.05 CCR.OA.B	Determining the total amount of money when each person pays an equal fee
2.OA.8 MWOTL	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? + 3, 6 × 6 =?	CC.3.OA.4 ESS01.03.05 CCR.OA.B	Determining a quantity of items per box or the total of items knowing how many total items there are OR how many boxes of inventory there are.
Understand properties of multiplication and the relationship between multiplication and division.			
2.OA.9	Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication). 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication). Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property). Students need not use formal terms for these properties.	CC.3.OA.5 ESS01.03.02 CCR.OA.B	Counting cash in varying denominations (e.g., 2 ten-dollar bills = $20 2 fifty-dollar bills per 6 family members = 2 × 50 × 6 = 50 ×6 × 2 4 twenty-dollar bills and 7 more twenty-dollar bills is the same as 11 twenty-dollar bills
2.OA.10 MWOTL	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.	CC.3.OA.6 ESS01.03.02 CCR.OA.B	Working out how many cars are needed to transport a group of people

Multiply and divide within 100.			
2.OA.11	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 ×5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. Know from memory all products of two one-digit numbers.	CC.3.OA.7 ESS01.03.02 CCR.OA.B	Dividing work time for employees
Solve problems involving the four operations, and identify and explain patterns in arithmetic.			
2.OA.12	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (order of operations).	CC.3.OA.8 ESS01.03.02 ESS01.03.05 CCR.OA.B	Estimating amount of purchase to nearest 10 dollars Estimating distances between cities Purchasing tickets for a group of people Ex. Ricardo used half a box of nails to build a desk. One box contains 20 nails. If Ricardo plans to build 5 more desks, how many boxes of nails will he need?
2.OA.13	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	CC.3.OA.9 ESS01.03.02 ESS01.03.03 CCR.OA.B	Laying tile on a floor Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

NUMBER AND OPERATIONS IN BASE TEN (NBT)			
Understand place value.			
2.NBT.1 MWOTL	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (e.g., 706 equals 7 hundreds, 0 tens, and 6 ones). Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	CC.2.NBT.1 ESS01.03.01 CCR.NBT.B	Exchanging money into small bills or coins
2.NBT.2 MWOTL	Count within 1000; skip-count by 5s, 10s, and 100s.	CC.2.NBT.2 ESS01.03.01 CCR.NBT.B	Reading the axis scale of a graph accurately
2.NBT.3 MWOTL	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	CC.2.NBT.3 ESS01.03.01 CCR.NBT.B	Writing a rent or tuition check
2.NBT.4 MWOTL	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	CC.2.NBT.4 ESS01.03.03 CCR.NBT.B	Monitoring a bank account Comparing food calories Reading nutritional information on foods
Use place value understanding and properties of operations to add and subtract and to perform multi-digit arithmetic.			
2.NBT.5	Add up to four two-digit numbers using strategies based on place value and properties of operations.	CC.2.NBT.6 ESS01.03.02 CCR.NBT.B	Monitoring a bank account Finding the total cost of filling a gas tank in a month

2.NBT.6 MWOTL	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	CC.2.NBT.7 ESS01.03.02 CCR.NBT.B	Finding how much money is left after shopping at different places Counting the amount of inventory of an item separated between a storage area and the sales floor.
2.NBT.7 MWOTL	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	CC.2.NBT.8 ESS01.03.02 CCR.NBT.B	Ex. Martha earns $400 per week. If Pablo earns $100 less per week than Martha, how much money does Pablo earn per week?
2.NBT.8 MWOTL	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)	CC.2.NBT.9 ESS01.03.02 CCR.NBT.B	Ex. Solve 47+65 and explain the answer by drawing a diagram. Use blocks as manipulatives
2.NBT.9	Use place value understanding to round whole numbers to the nearest 10 or 100.	CC.3.NBT.1 CCR.NBT.B	Ex. A local bakery sold 216 pies in October. In November, the bakery sold 89 pies. To the nearest ten, about how many pies did the bakery sell in total?
2.NBT.10 MWOTL	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	CC.2.NBT.5 CC.3.NBT.2 ESS01.03.02 CCR.NBT.B	Ex. Sofia spent 17 minutes washing the dishes. She spent 38 minutes cleaning her living room. Explain how you can use mental math to find how long Sofia spent on the two tasks.

2.NBT.11 MWOTL	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	CC.3.NBT.3 ESS01.03.02 CCR.NBT.B	Ex. Amanda exercises for 50 minutes each day. How many minutes will she exercise in 7 days?
NUMBERS AND OPERATIONS IN FRACTIONS (NF)			
Develop understanding of fractions as numbers.			
2.NF.1 MWOTL	Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$.	CC.3.NF.1 ESS01.03.01 CCR.NF.B	Ex. Felix cut a pizza into 8 pieces and then ate one piece. What fraction of the pizza did Felix eat? Ex. Three roommates share a utility bill. What fraction of the bill does each pay?
2.NF.2 MWOTL	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{1}{b}$ on the number line.	CC.3.NF.2 ESS01.03.01 CCR.NF.B	Fractions of inches on a measuring tape.

2.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. b. Recognize and generate simple equivalent fractions (e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent (e.g., by using a visual fraction model). c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole.	CC.3.NF.3 ESS01.03.03 CCR.NF.B	Recording the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model) Recognize the following relationships: half of a gallon of milk, half of a dozen eggs, quarter of a year, quarter of a month
MEASUREMENT AND DATA (MD)			
Measure and estimate lengths in standard units.			
2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	CC.2.MD.1 ESS01.03.04	Measuring a room for new carpet
2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	CC.2.MD.2 ESS01.03.04 CCR.MD.B	Sorting by size Use both the inch side and the centimeter side of a ruler to measure objects, such as a textbook, a calculator, one's hand span.
2.MD.3 MWOTL	Estimate lengths using units of inches, feet, centimeters, and meters.	CC.2.MD.3 ESS01.03.04 CCR.MD.B	Hanging a picture or an award on the wall

2.MD.4 MWOTL	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	CC.2.MD.4 ESS01.03.03 CCR.MD.B	Measure a photo and a frame to determine the needed width of a mat.
Relate addition and subtraction to length.			
2.MD.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem).	CC.2.MD.5 ESS01.03.02 ESS01.03.04	Ex. A dog kennel is 6 feet long. The space for the kennels at a shelter is 34 feet. How many kennels will fit in this space?
2.MD.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2... and represent whole-number sums and differences within 100 on a number line diagram.	CC.2.MD.6 ESS01.03.01 CCR.MD.B	Create a number line with whole number intervals
Work with time and money.			
2.MD.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	CC.2.MD.7	Computing hours worked or pay for babysitter
2.MD.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.	CC.2.MD.8 ESS01.03.02	Ex. If you have 2 dimes and 3 pennies, how many cents do you have?
Represent and interpret data.			
2.MD.9	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.	CC.2.MD.10 CC.3.MD.3 ESS01.03.06 CCR.MD.B	Draw a bar graph in which each square in the bar graph might represent 5 people.
2.MD.10	Generate measurement data by measuring lengths of several objects using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units - whole numbers, halves, or quarters.	CC.2.MD.9 CC.3.MD.4 ESS01.03.04 CCR.MD.B	

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.			
2.MD.11 MWOTL	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., by representing the problem on a number line diagram).	CC.3.MD.1 ESS01.03.01 ESS01.03.05 CCR.MD.B	Checking bus schedules in a.m. and p.m.
2.MD.12 MWOTL	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings such as a beaker with a measurement scale to represent the problem).	CC.3.MD.2 ESS01.03.02 CCR.MD.B	Following a recipe Administering a dose of medicine
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.			
2.MD.13 MWOTL	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	CC.3.MD.5 ESS 01.03.04 CCR.MD.B	Buying carpet tiles, tile for a bathroom or wallpaper
2.MD.14 MWOTL	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	CC.3.MD.6 ESS 01.03.04 CCR.MD.B	Counting the tiles on a floor

<table>
<tr><td>2.MD.15

MWOTL</td><td>Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</td><td>CC.3.MD.7
ESS 01.03.02
ESS 01.03.04
CCR.MD.B</td><td>Carpeting an odd shaped room

Designing and laying bricks for a patio</td></tr>
<tr><td colspan="4">Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</td></tr>
<tr><td>2.MD.16</td><td>Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</td><td>CC.3.MD.8
ESS01.03.04
ESS01.03.05
CCR.MD.B</td><td>Figuring the amount of molding needed around a window

An electronic dog fence is 500 ft long. If a rectangular yard is 100 feet long, how wide can it be and still be enclosed by the dog fence?</td></tr>
</table>

GEOMETRY (G)			
Reason with shapes and their attributes.			
2.G.1 MWOTL	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. Sizes are compared directly or visually, not compared by measuring.	CC.2.G.1 ESS01.03.03 CCR.G.B	Creating a pattern for laying tiles Matching patterns for home decorating by design and shape.
2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	CC.2.G.2 ESS01.03.02	Making flash cards
2.G.3 MWOTL	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	CC.2.G3 CCR.G.B	Cutting a pie or cake into equal parts
2.G.4	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	CC.3.G.1 CCR.G.B	Drawing and sculpting objects with clay Make a Venn diagram or another type of diagram to list the properties of different quadrilaterals
2.G.5 MWOTL	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.	CC.3.G.2 ESS01.03.02 CCR.G.B	Splitting a pizza or pie into equal slices

NRS Level 3 Overview

Operations and Algebraic Thinking (OA)

- Use the four operations with whole numbers to solve problems
- Gain familiarity with factors and multiples
- Generate and analyze patterns
- Write and interpret numerical expressions
- Analyze patterns and relationships

Expressions and Equations (EE)

- Apply and extend previous understandings of arithmetic to algebraic expressions
- Reason about and solve one-variable equations and inequalities
- Represent and analyze quantitative relationships between dependent and independent variables

Number and Operations in Base Ten (NBT)

- Generalize place value understanding for multi-digit whole numbers
- Use place value understanding and properties of operations to perform multi-digit arithmetic
- Understand the place value system
- Perform operations with multi-digit whole numbers and with decimals to hundredths

The Number System (NS)

- Compute fluently with multi-digit numbers

Number and Operations – Fractions (NF)

- Extend understanding of fraction equivalence and ordering
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers
- Understand decimal notation for fractions, and compare decimal fractions
- Use equivalent fractions as a strategy to add and subtract fractions
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions

Ratios and Proportional Reasoning (RP)

- Understand ratio concepts and use ratio reasoning to solve problems.

Measurement & Data (MD)

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit
- Represent and interpret data
- Geometric measurement: understand concepts of angle and measure angles
- Convert like measurement units within a given measurement system
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition

Geometry (G)

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles
- Graph points on the coordinate plane to solve real-world and mathematical problems
- Classify two-dimensional figures into categories based on their properties
- Solve real-world and mathematical problems involving area, surface area, and volume

Statistics and Probability (SP)

- Develop understanding of statistical variability
- Summarize and describe distributions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 3 – Low Intermediate Basic Education
(Grade Levels 4 – 5)

At the Low Intermediate Basic Education Level, instructional time should focus on six critical areas[18]:

1. Developing understanding and fluency with multi-digit multiplication, and of dividing to find quotients involving multi-digit dividends;
2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers;
3. Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions);
4. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations;
5. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry; and
6. Developing understanding of volume.

(1) Adult learners generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Adult learners develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $\frac{15}{9} = \frac{5}{3}$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the

[18] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Adult learners apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(4) Adult learners develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(5) Adult learners describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

(6) Adult learners recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical understanding and expertise.

NRS Level 3 – Low Intermediate Basic Education
(Grade Levels 4 – 5)

Standard Number	MATH STANDARD	Reference	Example task, problem, or activity
OPERATIONS AND ALGEBRAIC THINKING (OA)			
Use the four operations with whole numbers to solve problems.			
3.OA.1 MWOTL	Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.	CC.4.OA.1 ESS01.03.02 CCR.OA.C	Calculating the total number of items in batches Ex. Warehouse employees must pack a minimum of 85 boxes per shift. Sheila packed four times the minimum. How many boxes did she pack?
3.OA.2	Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison.	CC.4.OA.2 ESS01.03.02 ESS01.03.05 CCR.OA.C	Planning a neighborhood party Ex. Sheila delivered 3 times as many packages as Carlos. Together, they delivered 57 packages. How many packages did Sheila deliver?
3.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	CC.4.OA.3 ESS01.03.02 ESS01.03.05 CCR.OA.C	Planning what kind of pizza or sandwiches to order for an employee luncheon Ex. A carpenter has a board that is 10 feet long. He wants to make 6 table legs that are all the same length. What is the longest each leg can be?

Gain familiarity with factors and multiples.			
3.OA.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	CC.4.OA.4 ESS01.03.02 CCR.OA.C	Figuring how many ways change from $100 can be given/made Ex. A food bank has 50 cans of vegetables, 30 loaves of bread, and 100 bottles of water. The volunteers will put the items into boxes. Each box will have the same number of food items and all the items in the box will be the same type. How many items can they put in each box?
3.OA.5	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).	CC.6.NS.4 CCR.NS.C	Ex. A store clerk is bagging spices. He has 18 teaspoons of cinnamon and 30 teaspoons of nutmeg. Each bag needs to contain the same number of teaspoons, and each bag can contain only one spice. How many teaspoons of spice should the clerk put in each bag? How many bags of each spice will there be?

Generate and analyze patterns.			
3.OA.6 MWOTL	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	CC.4.OA.5 ESS01.03.03 CCR.OA.C	Designing a necklace and describing the assembly rule Ex. An artist is arranging tiles in rows to decorate a wall. Each new row has 2 fewer tiles than the row below it. If the first row has 23 tiles, how many tiles will be in the fifth row?
Write and interpret numerical expressions.			
3.OA.7 MWOTL	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	CC.5.OA.1 ESS01.03.04 CCR.OA.C	Entering an expression/formula in a spreadsheet Ex. Juanita has a cafe. Each day, she bakes 24 muffins. She gives away 3 and sells the rest. Each day, she also bakes 36 cupcakes. She gives away 4 and sells the rest. Write and evaluate an expression to represent the total number of muffins and cupcakes Juanita sells in 5 days.

3.OA.8 MWOTL	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.	CC.5.OA.2 ESS01.03.03 CCR.OA.C	Creating and solving word problems Ex. Rashad had 25 scratch-off lottery tickets. He shared them equally among himself and 4 coworkers. Then Rashad found 2 more tickets in his pocket. Write an expression to match the situation.
Analyze patterns and relationships.			
3.OA.9	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	CC.5.OA.3 ESS01.03.06	Students determine the rule of a simple pattern using a table with missing numbers, then create a graph Ex. Consider a recipe in which you need 2 eggs and 3 cups of flour. Make ordered pairs for the number of eggs and the number of cups of flour related to the multiple of the recipe you are using (doubling, tripling, etc.) Graph these ordered pairs and describe the relationship between the two sets of coordinates.

EXPRESSIONS AND EQUATIONS (EE)			
Apply and extend previous understandings of arithmetic to algebraic expressions.			
3.EE.1 MWOTL	Write and evaluate numerical expressions involving whole-number exponents	CC.6.EE.1 CCR.EE.C	Entering an expression/formula in a spreadsheet Ex. Karina wants to strengthen her arms so she doubles the number of push-ups she does each day. On the fifth day, she does 2 × 2 × 2 × 2 × 2 push-ups. Use an exponent to write an expression for the number of push-ups Karina does on the fifth day and then evaluate the expression Ex. A scientist observes that his population of fruit flies doubles every 2 weeks. If he begins with 2 flies in week 1, in which week will he have 128 flies?

3.EE.2 MWOTL	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (order of operations).	CC.6.EE.2 ESS01.03.04 CCR.EE.C	Keeping personal finance records Ex. a. A company rents bicycles for a fee of \$10 plus \$4 per hour of use. Write an algebraic expression for the total cost in dollars for renting a bicycle for h hours. Ex. b. The formula c = 5(f - 32) ÷ 9 gives the Celsius temperature in *c* degrees for a Fahrenheit temperature of *f* degrees. What is the Celsius temperature for a Fahrenheit temperature of 122 degrees? Ex. c. Use the formulas V = s^3 and A = $6s^2$ to find the volume and surface area of a cube with sides of length s = $\frac{1}{2}$.

3.EE.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6 (4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.	CC.6.EE.3 ESS01.03.04 CCR.EE.C	Predicting pay on a base wage with tips Ex. Riley's parents got a cell phone plan that has a $40 monthly fee for the first phone. For each extra phone, there is a $15 phone service charge and a $10 text service charge. The expression 40 + 15e + 10e represents the total phone bill in dollars, where *e* is the number of extra phones. Simplify the expression by combining like terms.
3.EE.4	Identify when two expressions are equivalent For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.	CC.6.EE.4 CCR.EE.C	Entering an expression in multiple spreadsheets to produce a similar result Ex. Are 11(p + q) and 11p + (7q + 4q) equivalent?
Reason about and solve one-variable equations and inequalities.			
3.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	CC.6.EE.5 CCR.EE.C	Projecting whether certain budgets will work Ex. The inequality m ≤ $540 represents the amount of money that Kanwar can afford to spend on a new phone. He found a phone that costs $600; can he afford it? What about $540? $470?

3.EE.6 MWOTL	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.	CC.6.EE.6 ESS01.03.04 CCR.EE.C	Keeping personal finance records Ex. An architect is designing a building where each floor will be 12 feet tall. Write an expression for the number of floors the building can have for a given building height.
3.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	CC.6.EE.7 ESS01.03.05 CCR.EE.C	Ex. A business has variable costs and fixed costs. Total costs are the sum of fixed and variable costs. If Total costs for Marge's Candies is $4750 one month, and her fixed costs are $1300, what were her variable costs? Ex. The revenue Marge can expect from her Super Bon Bons is the product of the price and the number sold. If Marge's revenue from her Super Bon Bons one week was $216, and the price per pound was $3.50, how many pounds of Super Bon Bons did Marge sell that week?
3.EE.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	CC.6.EE.8 ESS01.03.03 CCR.EE.C	Ex. The inequality $x \leq 2$ represents the elevation x of a certain object found at a dig site. Graph the solutions of the inequality on the number line

Represent and analyze quantitative relationships between dependent and independent variables.			
3.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	CC.6.EE.9 ESS01.03.04 CCR.EE.C	Ex. In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.
NUMBER AND OPERATIONS IN BASE TEN (NBT)			
Generalize place value understanding for multi-digit whole numbers.			
3.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.	CC.4.NBT.1 ESS01.03.01 ESS01.03.02 CCR.NBT.C	Monitoring a bank account Determining a total bill or expense Ex. An apple orchard sells apples in bags of 10. The orchard sold a total of 2,430 apples one day. How many bags of apples was this?
3.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	CC.4.NBT.2 ESS01.03.03 CCR.NBT.C	Comparing numbers written in an article vs. on a graph. Ex. An online newspaper had 350,080 visitors in October, 350,489 visitors in November, and 305,939 visitors in December. What is the order of the months from greatest to least number of visitors?

3.NBT.3	Use place value understanding to round multi-digit whole numbers to any place.	CC.4.NBT.3 CCR.NBT.C	Ex. An online newspaper had 350,080 visitors in October, 350,489 visitors in November, and 305,939 visitors in December. What is the order of the months from greatest to least number of visitors?
Use place value understanding and properties of operations to perform multi-digit arithmetic.			
3.NBT.4 MWOTL	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	CC.4.NBT.4 ESS01.03.02 CCR.NBT.C	Calculating downs and distance in football Ex. House A is 1,840 square ft, House B is 2,060 square ft. How much larger is House B than House A?
3.NBT.5 MWOTL	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	CC.4.NBT.5 ESS01.03.02 CCR.NBT.C	Checking delivery of goods in small batches Ex. Jose runs 2 miles three times per week. How many miles does Jose walk in a week/ year?
3.NBT.6 MWOTL	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	CC.4.NBT.6 ESS01.03.02 CCR.NBT.C	Finding price of 2 cartons of milk or 6 bottles of soda Ex. There are 132 guests attending a community event. If 8 guests can fit in a row, how many full rows of guests can be made? How many guests are in the row that is not full?

Understand the place value system.			
3.NBT.7	Recognize that in a multi-digit number, a digit in one place represents ten times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.	CC.5.NBT.1 ESS01.03.01 CCR.NBT.C	Use a place-value chart and patterns to write numbers that are 10 times as much as or $\frac{1}{10}$ of any given number.
3.NBT.8	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.	CC.5.NBT.2 ESS01.03.02 CCR.NBT.C	Ex. Three friends are selling items at a farmer's market. Felipe makes $23.25 selling bread. Inez sells gift baskets and makes 100 times as much as Felipe. Carolyn sells pies and makes one tenth of the money Inez makes. How much money does each friend make?
3.NBT.9 MWOTL	Read, write, and compare decimals to thousandths.	CC.5.NBT.3 ESS01.03.01 CCR.NBT.C	Compare Olympic medalists' winning times in a variety of sports.
3.NBT.10 MWOTL	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.	CC.5.NBT.3a ESS01.03.01 ESS01.03.02 CCR.NBT.C	Write 347.392 in expanded form. $3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000}) = 347.392$
3.NBT.11 MWOTL	Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	CC.5.NBT.3b ESS01.03.03 CCR.NBT.C	Using Dewey Decimal System in a library Compare 1.567 to 1.562

3.NBT.12	Use place value understanding to round decimals to any place.	CC.5.NBT.4 ESS01.03.01 CCR.NBT.C	Understanding price tags or prices on a menu Ex. A machinist uses a tool to measure the diameter of a small pipe. The tool reads 0.276 inch. The sales department needs the decimal to the nearest tenth for the advertising chart. What number should be reported to the sales department?
Perform operations with multi-digit whole numbers and with decimals to hundredths.			
3.NBT.13 MWOTL	Fluently multiply multi-digit whole numbers using the standard algorithm.	CC.5.NBT.5 ESS01.03.02 CCR.NBT.C	Calculating attendance over multiple events at one location Ex. Last month, a manufacturing company shipped 452 boxes of brake pads. If each box contains 48 pads, how many brake pads did the company ship last month?
3.NBT.14 MWOTL	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	CC.5.NBT.6 ESS01.03.02 CCR.NBT.C	Calculating miles per gallon attained by a vehicle Ex. A factory processes 1,560 ounces of olive oil per hour. The oil is packaged into 24-ounce bottles. How many bottles does the factory fill in one hour?

3.NBT.15 MWOTL	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	CC.5.NBT.7 ESS01.03.02 CCR.NBT.C	Counting and recording total value of an item purchased on sale Ex. Alexia raises \$75.23 for a charity. Sue raises 3 times as much as Alexia. Manuel raises \$85.89. How much money do the three friends raise for the charity in all?
The Number System (NS)			
Compute fluently with multi-digit numbers			
3.NS.1 MWOTL	Fluently divide multi-digit numbers using the standard algorithm.	CC.6.NS.2 CCR.NS.C	Ex. A van is carrying 486 pounds. There are 27 boxes in the van. What is the average weight of each box in the van?
3.NS.2 MWOTL	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	CC.6.NS.3 CCR.NS.C	Using a four function calculator to find hourly rate given weekly pay or vice versa Ex. Gloria worked for 6 hours a day for 2 days at the bank and earned \$114.24. How much did she earn per hour?

NUMBER & OPERATIONS – FRACTIONS (NF)			
Extend understanding of fraction equivalence and ordering.			
3.NF.1 MWOTL	Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	CC.4.NF.1 ESS01.03.04 ESS01.03.05 CCR.NF.C	In recipe conversions, determining equal amounts Ex. Jamal delivered $\frac{5}{6}$ of his packages, Margaret delivered $\frac{3}{4}$ of hers, and Gerald delivered $\frac{10}{12}$ of his. Use visuals to show which two workers delivered the same number of packages.
3.NF.2 MWOTL	Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.	CC.4.NF.2 ESS01.03.03 CCR.NF.C	In recipe conversions, determining equal amounts (e.g., tablespoons to cups and cups to quarts) Compare recipe ingredient amounts between recipes for the same item (e.g., cookies)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.			
3.NF.3 MWOTL (c & d)	Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions by using a visual fraction model. c. Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction). d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem.	CC.4.NF.3 ESS01.03.02 ESS01.03.03 ESS01.03.04 CCR.NF.C	Ex. a. Jake ate $\frac{4}{8}$ of a pizza. Millie ate $\frac{3}{8}$ of the same pizza. How much of the pizza was eaten by Jake and Millie? Ex. b. $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8}$ or $\frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. Ex. c. James wants to send two gifts by mail. One package weighs $2\frac{3}{4}$ pounds. The other package weighs $1\frac{3}{4}$ pounds. What is the total weight of the packages? Ex. d. Mr. Warren uses $2\frac{1}{4}$ bags of mulch for his garden and another $4\frac{1}{4}$ bags for his front yard. He also uses $\frac{3}{4}$ bag around a fountain. How many total bags of mulch does Mr. Warren use?

3.NF.4 MWOTL	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. c. Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem.	CC.4.NF.4 ESS01.03.02 ESS01.03.04 CCR.NF.C	Ex. a. Use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \left(\frac{1}{4}\right)$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \left(\frac{1}{4}\right)$. Ex. b. Use a visual fraction model to express $3 \times \left(\frac{2}{5}\right)$ as $6 \times \left(\frac{1}{5}\right)$, recognizing this product as $\frac{6}{5}$. Ex. c. If each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
Understand decimal notation for fractions, and compare decimal fractions.			
3.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.	CC.4.NF.5 ESS01.03.01	Ex. Express $\frac{3}{10}$ as $\frac{30}{100}$ and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.
3.NF.6	Use decimal notation for fractions with denominators 10 or 100.	CC.4.NF.6 ESS01.03.01 CCR.NF.C	Understanding how the scale works at the deli counter Ex. Rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

3.NF.7	Compare two decimals to the hundredths place by reasoning about their size. Recognize that comparisons are valid only when two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).	CC.4.NF.7 ESS01.03.03 CCR.NF.C	Reading and comparing gas prices Reading and comparing metric measurements Ex. Tyrrell's commute to work is 3.47 miles and Shayna's commute is 3.4 miles. Whose commute is longer?
Use equivalent fractions as a strategy to add and subtract fractions.			
3.NF.8 MWOTL	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	CC.5.NF.1 ESS01.03.01 ESS01.03.02 CCR.NF.C	Understanding how much food is left after a party Ex. Hassan made a vegetable salad with $2\frac{3}{8}$ pounds of tomatoes, $1\frac{1}{4}$ pounds of asparagus, and $2\frac{7}{8}$ pounds of potatoes. How many pounds of vegetables did he use altogether?
3.NF.9	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ by observing that $\frac{3}{7}<\frac{1}{2}$.	CC.5.NF.2 ESS01.03.04 CCR.NF.C	Understanding how to read a digital scale when placing a fraction order at the deli For example, recognize an incorrect result $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ by observing that $\frac{3}{7}<\frac{1}{2}$. Ex. Uma started her trip with $1\frac{1}{2}$ gallons of gas in her car's gas tank. She bought an additional $6\frac{4}{5}$ gallons on her trip and arrived back home with $3\frac{3}{10}$ gallons left. How much gas did she use on the trip?

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.			
3.NF.10 MWOTL	Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}$ = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people, each person has a share of size $\frac{3}{4}$.	CC.5.NF.3 ESS01.03.03 ESS01.03.04 CCR.NF.C	Planning what kind of pizza or sandwiches to order for an employee luncheon Ex. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
3.NF.11 MWOTL	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $\left(\frac{a}{b}\right) \times q$ as *a* parts of a partition of q into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. For example, use a visual fraction model to show $\left(\frac{2}{3}\right)$ x 4 = $\frac{8}{3}$, and create a story context for this equation. Do the same with $\left(\frac{2}{3}\right)$ x $\left(\frac{4}{5}\right)$ = $\frac{8}{15}$. (In general, $\left(\frac{a}{b}\right)$ x $\left(\frac{c}{d}\right)$ = $\frac{ac}{bd}$.) b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	CC.5.NF.4 ESS01.03.04 CCR.NF.C	Ex. a. The gas tank of Mr. Tanaka's car holds 15 gallons of gas. He used $\frac{2}{3}$ of a tank of gas last week. How many gallons of gas did Mr. Tanaka use? Ex. b. Show the area of a rectangle with length $\frac{9}{10}$ inch and width $\frac{1}{2}$ inch by drawing a picture and multiplying the fractions.

3.NF.12	Interpret multiplication as scaling (resizing) by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b}$ = (n×a) / (n×b) to the effect of multiplying $\frac{a}{b}$ by 1.	CC.5.NF.5 ESS01.03.04 ESS01.03.05 CCR.NF.C	Ex. a. Fraser is making a scale drawing of a dog house. The dimensions of the drawing will be $\frac{1}{8}$ of the dimensions of the actual doghouse. The height of the actual doghouse is 36 $\frac{3}{4}$ inches. Will the dimensions of Fraser's drawing be equal to, greater than, or less than the dimensions of the actual dog house? Ex. b. Ricardo bikes 2 $\frac{1}{4}$ miles per hour. His friend bikes 1 $\frac{1}{3}$ times as far per hour. Who bikes more miles per hour?
3.NF.13	Solve real world problems involving multiplication of fractions and mixed numbers using visual fraction models or equations to represent the problem.	CC.5.NF.6 ESS01.03.04 CCR.NF.C	Ex. Kuman can carry 6 $\frac{1}{4}$ pounds of wood in from the warehouse to his truck. His father can carry 1 $\frac{1}{2}$ times as much wood as Kuman. How many pounds can Kuman's father carry?

3.NF.14 MWOTL a & b only	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to explain that $\left(\frac{1}{3}\right) \div 4 = \frac{1}{12}$ because $\left(\frac{1}{12}\right) \times 4 = \frac{1}{3}$. b. Interpret division of a whole number by a unit fraction, and compute such quotients. Use the relationship between multiplication and division to explain that $4 \div \left(\frac{1}{5}\right) = 20$ because $20 \times \left(\frac{1}{5}\right) = 4$. c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions using visual fraction models and equations to represent the problem.	CC.5.NF.7 ESS01.03.04 CCR.NF.C	Ex. a. Gianna cuts a half of a loaf of bread into 4 equal parts. What fraction of the whole loaf of bread does each of the 4 parts represent? Ex. b. A landscaper had 5 tons of rock to build decorative walls. He used $\frac{1}{4}$ ton of rock for each wall. How many decorative walls did he build? Ex. c. How much chocolate will each person get if 3 people share a $\frac{1}{2}$ pound of chocolate equally?

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.			
3.NF.15 MWOTL	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions using visual fraction models and equations to represent the problem. Use the relationship between multiplication and division to explain that $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$.	CC.6.NS.1 ESS01.03.04 CCR.NS.C	Preparing a recipe for a smaller group than the intended group Ex. How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? Ex. How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? Ex. How wide is a rectangular strip of land with length $\frac{3}{4}$ mile and area $\frac{1}{2}$ square mile?
Ratios and Proportional Relationships (RP)			
Understand ratio concepts and use ratio reasoning to solve problems.			
3.RP.1 MWOTL	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	CC.6.RP.1 ESS01.03.04 CCR.RP.C	Calculating miles per gallon attained by a vehicle Ex. Create sentences like “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak”. OR “For every vote candidate A received, candidate C received nearly three votes”.

3.RP.2 MWOTL	Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio *a*:*b* with $b \neq 0$ (*b* not equal to zero), and use rate language in the context of a ratio relationship. (Expectations for unit rates in this grade are limited to non-complex fractions.)	CC.6.RP.2 ESS01.03.04 CCR.RP.C	Estimating travel time in hours based on distance and speed Ex. Mimi drove 112 miles in 2 hours. How many miles per hour did Mimi drive? Ex. Creating sentences like "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." OR "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."
MEASUREMENT & DATA (MD)			
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.			
3.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	CC.4.MD.1 ESS01.03.04	Following the growth in height or weight of babies, children, or plants Ex. If a dining table is 4 feet long, how many inches long is it? Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).

3.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	CC.4.MD.2 ESS01.03.06 CCR.MD.C	Ex. If there are 3 weeks and 12 days left in the year, how many total days are left? Ex. Juan drinks 2 liters of water after soccer practice. How many milliliters does Juan drink? Ex. Barbara left her house at 7:15am and drove 33 minutes to work. What time did she arrive?
3.MD.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems.	CC.4.MD.3 CCR.MD.C	Finding the length of fencing around a garden Ex. Vanessa needs to buy weather-stripping for three windows in her living room. Each window is 2 feet by 3 feet. How much weather-stripping does she need to purchase altogether so she has enough to go around each window?
Represent and interpret data.			
3.MD.4	Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	CC.4.MD.4 ESS01.03.06 CCR.MD.C	Calculating the size of a container required to hold items of various lengths From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection

3.MD.5	Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots.	CC.5.MD.2 ESS01.03.06	Ex. A clerk in a health food store makes bags of trail mix. The amount of trail mix in each bag is listed below. $\frac{1}{4}$ lb, $\frac{1}{4}$ lb, $\frac{3}{4}$ lb, $\frac{1}{2}$ lb, $\frac{1}{4}$ lb, $\frac{3}{4}$ lb, $\frac{3}{4}$ lb, $\frac{3}{4}$ lb, $\frac{1}{2}$ lb, $\frac{1}{4}$ lb, $\frac{1}{2}$ lb, $\frac{1}{2}$ lb. Complete a line plot of the data.
Geometric measurement: understand concepts of angle and measure angles.			
3.MD.6	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through in one-degree angles is said to have an angle measure of *n* degrees.	CC.4.MD.5 ESS01.03.04 CCR.MD.C	Compare the orbit of the Earth vs the Moon in distanced traveled in one day. Use a visual to show the angle measurement.
3.MD.7	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	CC.4.MD.6 ESS01.03.04 CCR.MD.C	Use a protractor to measure the angle of incline of a road or wheelchair ramp

3.MD.8	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).	CC.4.MD.7 ESS01.03.02 ESS01.03.04 CCR.MD.C	Ex. Two angles measure a total of 180 degrees. If one angle measures 43 degrees, what is the measure of the second angle? Ex. An astronomer was rotating her telescope. She wished to rotate it 76 degrees. Unfortunately, after it rotated 35 degrees, it malfunctioned and had to be serviced. After it was serviced, how many more degrees would the telescope need to be rotated?
Convert like measurement units within a given measurement system.			
3.MD.9	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step real world problems.	CC.5.MD.1 ESS01.03.04 CCR.MD.C	Doing home repairs and carpentry projects Ex. A plumber uses 16 inches of tubing to connect each washing machine in a laundry mat to the water source. He needs to install 18 washing machines. How many yards of tubing will he need for the job?

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.			
3.MD.10 MWOTL	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	CC.5.MD.3 CCR.MD.C	Ex. a. A box can hold 1,000 unit cubes that measure 1 inch by 1 inch by 1 inch. Describe the dimensions of the box using unit cubes. Ex. b. A manufacturer ships its product in boxes with edges of 4 inches long. If 12 boxes are put in a carton and completely fill the carton, what is the volume of the carton?
3.MD.11 MWOTL	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	CC.5.MD.4 ESS01.03.04 CCR.MD.C	Hands on activity involving unit cubes of different measures, filling 3D figures of different sizes.

3.MD.12 MWOTL	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent three-fold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Apply the formulas V = (l)(w)(h) and V = (b)(h) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	CC.5.MD.5 ESS01.03.02 ESS01.03.04 CCR.MD.C	Determining the amount of sand needed to fill a sandbox Ex. a. Amanda's jewelry box is in the shape of a cube that has 6-inch edges. What is the volume of Amanda's jewelry box? Determining the amount of mulch needed to complete a landscaping project Ex. b. A construction company is digging a rectangular hole for a swimming pool. The hole will be 12 yards long, 7 yards wide, and 3 yards deep. How many cubic yards of dirt will the company need to remove? Determining the amount of water needed to fill a pool Ex. c. Roman is installing an L-shaped garden box in his backyard. One box is 7 feet long and 3 feet wide. The other box is 4 feet long and 3 feet wide. Both boxes are 2 feet tall. What is the total volume of Roman's L-shaped garden box?

GEOMETRY (G)			
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.			
3.G.1 MWOTL	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	CC.4.G.1 CCR.G.C	Draw a map of a neighborhood Identify lines and angles on a map
3.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	CC.4.G.2	Sorting shapes based on type of line, type of angle, etc.
3.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	CC.4.G.3	Creating holiday designs for greeting cards or crafts Identify symmetry in blueprints, mandalas, snowflakes, etc.
Graph points on the coordinate plane to solve real-world and mathematical problems.			
3.G.4 MWOTL	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	CC.5.G.1 ESS01.03.04 CCR.G.C	Use graph paper to draw and label a coordinate plane

3.G.5 MWOTL	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	CC.5.G.2 ESS01.03.04 CCR.G.C	Graph distance traveled by each person in a group Graph growth or time data presented in the news
Classify two-dimensional figures into categories based on their properties.			
3.G.6 MWOTL	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	CC.5.G.3 CCR.G.C	Sorting shapes of all sizes into sub-categories and explaining why
3.G.7	Classify two-dimensional figures in a hierarchy based on properties.	CC.5.G.4	Create a 2D shape classification chart
Solve real-world and mathematical problems involving area, surface area, and volume.			
3.G.8 MWOTL	Find area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	CC.6.G.1 CCR.G.C	Laying tile on a floor Ex. Stu is making a stained glass window in the shape of a regular pentagon. The pentagon can be divided into congruent triangles, each with a base of 8.7 inches and a height of 6 inches. What is the area of the window?
3.G.9	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = *l w h* and V = *b h* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	CC.6.G.2	Filling a sandbox with sand or a garden with mulch Ex. A shipping crate is shaped like a rectangular prism. It is $5\frac{1}{2}$ feet long by 3 feet wide by 3 feet high. What is the volume of the crate?

3.G.10 MWOTL	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	CC.6.G.3 CCR.G.C	Ex. A carpenter is making a custom shelf in the shape of a parallelogram. She begins by drawing parallelogram *RSTU* on a coordinate plane with vertices *R*(1, 0), *S*(-3, 0), and *T*(-2, 3). What are the coordinates of vertex *U*?
3.G.11	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	CC.6.G.4 CCR.G.C	Estimating the amount of fabric needed to cover a piece of furniture Ex. A gift box measures 14 in. by 12 in. by 6 in. How much wrapping paper is needed to exactly cover the box?
STATISTICS AND PROBABILITY (SP)			
Develop understanding of statistical variability.			
3.SP.1 MWOTL	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	CC.6.SP.1 CCR.SP.C	Understanding there are no “true” measures for statistical quantities (i.e., home values, expected wages, expected prices, expected profits or losses) Ex. “How old am I?” is not a statistical question, but “How old are the students in my class?” is a statistical question because one anticipates variability in students’ ages.

3.SP.2 MWOTL	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	CC.6.SP.2 CCR.SP.C	Analyze stock market, real estate sales, or holiday spending data
3.SP.3 MWOTL	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	CC.6.SP.3 CCR.SP.C	Recording information about students in class (i.e., age, number of children, number of years at current residence) Ex. Richard's science test scores are 76, 80, 78, 84, and 80. His math test scores are 100, 80, 73, 94, and 71. Compare the medians and interquartile ranges.
Summarize and describe distributions.			
3.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	CC.6.SP.4 ESS01.03.06 CCR.SP.C	Tracking one's daily expenses Collect data (daily expenses, heights of classmates, temperatures for 2 weeks, etc.) and display in various forms

NRS Level 4 Overview

Ratios and Proportional Relationships (RP)

- Understand ratio concepts and use ratio reasoning to solve problems
- Analyze proportional relationships and use them to solve real-world and mathematical problems

The Number System (NS)

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- Compute fluently with multi-digit numbers and find common factors and multiples
- Apply and extend previous understandings of numbers to the system of rational numbers
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
- Know that there are numbers that are not rational, and approximate them by rational numbers

Expressions and Equations (EE)

- Use properties of operations to generate equivalent expressions
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations
- Work with radicals and integer exponents
- Understand the connections between proportional relationships, lines, and linear equations
- Analyze and solve linear equations and pairs of simultaneous linear equations

Functions (F)

- Define, evaluate, and compare functions
- Use functions to model relationships between quantities

NRS Level 4 Overview (continued)

Geometry (G)

- Draw, construct, and describe geometrical figures and describe the relationship between them
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
- Understand congruence and similarity using physical models, transparencies, or geometry software
- Understand and apply the Pythagorean Theorem
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

Statistics and Probability (SP)

- Summarize and describe distributions
- Investigate chance processes and develop, use, and evaluate probability models
- Investigate patterns of association in bivariate data

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 4 – Middle Intermediate Basic Education
(Grade Levels 6 – 7)

At the High Intermediate Basic Education Level, instructional time should focus on seven critical areas[19]:

1. Developing understanding of and applying proportional relationships.
2. Developing understanding of operations with rational numbers and working with expressions and linear equations.
3. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.
4. Grasping the concept of a function and using functions to describe quantitative relationships.
5. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.
6. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.
7. Drawing inferences about populations based on samples, they build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations.

(1) Adult learners extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. They use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Learners solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects; they also graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Adult learners develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Learners extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), they explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

[19] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

(3) Adult learners use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Learners also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At the conclusion of Level 4, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Adult learners strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Learners solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. They use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(4) Adult learners grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representation.

(5) Adult learners continue their work on solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Learners work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(6) Adult learners use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. They show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Learners understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. Finally, learners apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

(7) Adult learners begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical understanding and expertise.

NRS Level 4 – Middle Intermediate Basic Education (Grade Levels 6-7)

<table>
<tr><th>Standard Number</th><th>MATH STANDARD</th><th>Reference</th><th>Example task, problem, or activity</th></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)</td></tr>
<tr><td colspan="4">Understand ratio concepts and use ratio reasoning to solve problems.</td></tr>
<tr><td>4.RP.1

MWOTL</td><td>Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).
a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed.
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td><td>CC.6.RP.3
ESS01.03.06
CCR.RP.D</td><td>Ex. a. Helen walks at a rate of 3 miles in 1 hour. Write this ordered pair and construct a table with additional equivalent pairs. Then plot on a coordinate plane.

Ex. b. If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Ex. c. The utilities bill for the Lopezes' home in April was $132. Forty-two percent of the bill was for gas, and the rest was for electricity. How much did the Lopezes pay for gas, and how much did they pay for electricity?

Ex. d. The 102nd floor of a skyscraper in Chicago is the highest occupied floor. It is 1,431 feet above the ground. How many yards above the ground is the 102nd floor?</td></tr>
</table>

Analyze proportional relationships and use them to solve real-world and mathematical problems.			
4.RP.2 MWOTL	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	CC.7.RP.1 ESS01.03.04 ESS01.03.05 CCR.RP.D	Mixing various quantities of cleaning fluids based on one set of directions. Ex. If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\left(\frac{1}{2}\right)/\left(\frac{1}{4}\right)$ miles per hour, equivalent to 2 miles per hour.
4.RP.3 MWOTL	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost *t* is proportional to the number *n* of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as *t* = *pn*. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	CC.7.RP.2 ESS01.03.06 CCR.RP.D	Ex. a.-d. In January, Alejandra signed up for a membership at Anytime Fitness. The plan she chose cost $95 in start-up fees and then $20 per month starting in February. Edwin also signed up at Anytime Fitness in January. His plan cost $35 per month starting in February, and his start-up fees were waived. Create tables for both Alejandra and Edwin comparing the cost for each of 12 months, plot the data of each table on a graph, decide if either or both gym memberships are described by a proportional relationship, and find an equation for one or both memberships.

4.RP.4 MWOTL	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	CC.7.RP.3 ESS01.03.02 CCR.RP.D	Determining the savings when buying something 50% off Determining the difference in tip at 15% or 20% of bill
THE NUMBER SYSTEM (NS)			
Apply and extend previous understandings of numbers to the system of rational numbers.			
4.NS.1	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	CC.6.NS.5 CCR.NS.D	Understanding wind chill information Analyze debit/credit card statements, weather reports, etc.)
4.NS.2	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous levels to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself (e.g., –(–3) = 3, and that 0 is its own opposite). b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	CC.6.NS.6 CCR.NS.D	Ex. a. Jacob needs to graph the opposite of -6 on a horizontal number line. Should he graph it to the left or right of 0? Ex. b. A town's post office is located at the point (7, 5) on a coordinate plane. In which quadrant is the post office located? Ex. c. Use a coordinate plane map to identify locations of places and plot locations for new establishments being built.

4.NS.3 MWOTL	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. d. Distinguish comparisons of absolute value from statements about order.	CC.6.NS.7 ESS01.03.03 ESS01.03.04 CCR.NS.D	Reading thermometers Ex. a. Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right Reading a bank balance Ex. b. Stan's bank account balance is less than -\$20.00 but greater than -\$21.00. What could Stan's account balance be? Monitoring a checking account Ex. c. For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. Ex. d. On Wednesday, Miguel's bank account balance was -\$55. On Thursday, his balance was less than that. Use absolute value to describe Miguel's balance on Thursday as a debt.

4.NS.4	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	CC.6.NS.8 CCR.NS.D	Tracking temperature and precipitation rates throughout the year Ex. A fire station is located 2 units east and 6 units north of a hospital. If the hospital is located at a point (-2, -3) on a coordinate map, what are the coordinates of the fire station?

<table>
<tr><th colspan="4">Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</th></tr>
<tr><td>4.NS.5

MWOTL</td><td>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

(NOTE: In NRS 4 the focus is on understanding the use of the number line for finding or verifying sums with positive and negative rational numbers, defining additive inverse, and rewriting subtraction problems as addition problems. This standard will be fully addressed in NRS 5.)</td><td>CC.7.NS.1
CCR.NS.D</td><td>Ex. Aakash, Bao Ying, Chris and Donna all live on the same street as their office building, which runs from east to west. Aakash lives $5\frac{1}{2}$ blocks to the west. Bao Ying lives $4\frac{1}{4}$ blocks to the east. Chris lives $2\frac{3}{4}$ blocks to the west. Donna lives $6\frac{1}{2}$ blocks to the east. Represent the relative position of the houses on a number line, with the office at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers. How far is each house from the office and from each other?</td></tr>
</table>

4.NS.6 MWOTL	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (NOTE: This standard will be fully addressed in NRS 5.)	CC.7.NS.2 ESS01.03.02 CCR.NS.D	Ex. a. & b. A water well drilling rig has dug to a height of –60 feet after one full day of continuous use. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours? If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running? Ex. c. Kylie and her friends are going on a hike and she is making trail mix to take. The recipe calls for 2 cups of peanuts, 1 cup of chocolate chips, 1 cup of dried cranberries, and $1\frac{1}{2}$ cups of pretzels. If she and three friends are going, how much of each ingredient will they get? Ex. d. Javier and 4 coworkers are driving to a conference and will split the cost of tolls. If the total cost of tolls for the entire trip is $8, how much will each person pay?

4.NS.7 MWOTL	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) (NOTE: This standard will be fully addressed in NRS 5.)	CC.7.NS.3 ESS01.03.02 CCR.NS.D	Ex. Three departments at Sunview Supermarket collected the most canned goods for a local fundraiser, and so they won a $600 prize to share among them. The produce department collected 3,760 cans, the meat department collected 2,301, and the dairy department collected 1,855. How should they divide the money so that each department gets the same fraction of the prize money as the fraction of the canned goods that they collected?
Know that there are numbers that are not rational, and approximate them by rational numbers.			
4.NS.8	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that neither repeat nor terminate. Know that other numbers are called irrational. (NOTE: This standard will be fully addressed in NRS 5.)	CC.8.NS.1	Identify numbers as rational or irrational and convert rational numbers into decimal form.
4.NS.9	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). (NOTE: This standard will be fully addressed in NRS 5.)	CC.8.NS.2 ESS01.03.06 CCR.NS.D	Ex. By truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

EXPRESSIONS AND EQUATIONS (EE)			
Use properties of operations to generate equivalent expressions.			
4.EE.1 MWOTL	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	CC.7.EE.1 CCR.EE.D	Finding a price increase of 10% Ex. Simplify the expression 7-2(3-8*x*)
4.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related	CC.7.EE.2 CCR.EE.D	Ex. a + 0.05*a* = 1.05*a* means that “increase by 5%” is the same as “multiply by 1.05. Ex. Malia is at an amusement park. She bought 14 tickets, and each ride requires 2 tickets. Write an expression that gives the number of tickets Malia has left in terms of *x*, the number of rides she has already gone on. Find at least one other expression that is equivalent to it.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.			
4.EE.3 MWOTL	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	CC.7.EE.3 ESS01.03.05 CCR.EE.D	Keeping personal finance récords Ex. If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. Ex. If you want to place a towel bar 9 $\frac{3}{4}$ inches long in the center of a door that is 27 $\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
4.EE.4 MWOTL	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	CC.7.EE.4 ESS01.03.05 CCR.EE.D	Keeping personal finance records or using a spreadsheet Ex. As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

Work with radicals and integer exponents.			
4.EE.5	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	CC.8.EE.1 CCR.EE.D	Ex. Write equivalent expressions for 3^2 x $3^{(-5)}$ Answers: $3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$
4.EE.6	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	CC.8.EE.2 CCR.EE.D	Ex. Evaluate $\sqrt{144}$ and $\sqrt[3]{143}$
4.EE.7	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	CC.8.EE.3 CCR.EE.D	Ex. Estimating the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determining that the world population is more than 20 times larger
4.EE.8	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	CC.8.EE.4 CCR.EE.D	Calculate distance between planets; measurement of small (microscopic) items (an ant's leg, a cell, etc.)
Understand the connections between proportional relationships, lines, and linear equations.			
4.EE.9 MWOTL	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	CC.8.EE.5 ESS01.03.06 CCR.EE.D	Comparing a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed
4.EE.10	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	CC.8.EE.6	Use a coordinate plane to observe triangles in roof trusses, tents, or other similar triangle forms.

Analyze and solve linear equations and pairs of simultaneous linear equations.			
4.EE.11 MWOTL	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	CC.8.EE.7 CCR.EE.D	Ex. a. Solve $-3x+5=20$ Ex. b. Solve $3(x+2)=x-18$
4.EE.12 MWOTL	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	CC.8.EE.8 CCR.EE.D	Ex. In order for Windy City Bicycles to stay in business they must make a profit. Each customer will pay $5 to reserve a bike and $12 per hour to rent the bike. Daily they have 10 customers. It costs the bike company $200 every day to maintain their rental space as well as $8 per hour per employee. They have 4 employees. How many bikes are needed to break even?

FUNCTIONS (F)			
Define, evaluate, and compare functions.			
4.F.1 MWOTL	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	CC.8.F.1 CCR.F.D	Ex. The number of goods produced each hour from a particular assembly line is graphed to monitor production. Does this graph represent a function?
4.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	CC.8.F.2 ESS01.03.06 CCR.F.IF.E	Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
4.F.3 MWOTL	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	CC.8.F.3 ESS01.03.06 CCR.F.D	Ex. The function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. Given a function table with *x* and *y* values, determine if the function is linear or nonlinear and write its equation.

Use functions to model relationships between quantities.			
4.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	CC.8.F.4 ESS01.03.06 CCR.F.D	Ex. Mina collects figurines. This year she began with 12 figurines, and acquired 3 new ones each month. Construct a graph for the year showing how many figurines she owns each month. Write an equation to calculate how many she will have after two years if she continues accumulating at this rate. After 3 years, Mina has 462 figurines in her collection and she is ready to collect something else. If she wants to sell the entire collection, how many must she sell each month to dispose of all of them in a year? Construct a graph showing this relationship and write an equation for the graph.

4.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	CC.8.F.5 ESS01.03.06 CCR.F.D	Reading a graph in an ad or poster Ex. Cherry's heart rate is illustrated on the given graph. At time 0, she was at rest (called the base rate). During which intervals was her heart rate increasing? How long did it take to return to base rate? During this time, Cherry was sitting quietly, walking, jumping rope, and doing yoga. Label different sections of the graph with when you think she was engaged in each of these activities.
GEOMETRY (G)			
Draw, construct, and describe geometrical figures and describe the relationships between them.			
4.G.1 MWOTL	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	CC.7.G.1 CCR.G.D	Buying carpet tiles or wallpaper Ex. Mariko has an 80:1 scale-drawing of the floor plan of her house. On the floor plan, the dimensions of her rectangular living room are $1\frac{7}{8}$ inches by $2\frac{1}{2}$ inches. What is the area of her real living room in square feet?

4.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	CC.7.G.2	Ex. Draw the following scenario: Judith walked $\frac{8}{10}$ mile south from her house to the store. Then she walked east from the store $1\frac{1}{2}$ miles to her friend Niko's house. After Niko's house, Judith walked northwest back home which took $1\frac{7}{10}$ miles.
4.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	CC.7.G.3	Slice a box, brick of cheese, carrot, etc. and describe the cross sections
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.			
4.G.4 MWOTL	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	CC.7.G.4 CCR.G.D	Calculate the circumference and area of a circle with a diameter of 24 inches Ex. A contractor is replacing a carousel floor. If the carousel is 50 feet in diameter, how much flooring will be needed?
4.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	CC.7.G.5 CCR.G.D	Cutting molding at a correct angle so that both ends meet with no space between
4.G.6 MWOTL	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	CC.7.G.6 ESS01.03.04 CCR.G.D	Buying paint for a project Ex. How much wood would be needed to construct a birdhouse (rectangular prism topped by triangular prism)?

Understand congruence and similarity using physical models, transparencies, or geometry software.			
4.G.7	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	CC.8.G.1	Use origami paper or other manipulatives to model the properties
4.G.8 MWOTL	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	CC.8.G.2 CCR.G.D	Use graph paper and tangrams to model congruence
4.G.9	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	CC.8.G.3	Use coordinate planes and tangrams to observe and notate the effects
4.G.10 MWOTL	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	CC.8.G.4 CCR.G.D	Use graph paper and tangrams to model similarity
4.G.11	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	CC.G.SRT.2	
4.G.12	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. (NOTE: This standard will be fully addressed in NRS 5.)	CC.8.G.5 CCR.G.D	Arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.			
4.G.13	Explain a proof of the Pythagorean Theorem and its converse.	CC.8.G.6	
4.G.14 MWOTL	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	CC.8.G.7 CCR.G.D	Calculating the distance from roof peak to gutter from the ground Ex. A 13 feet ladder is placed 5 feet away from a wall. The distance from the ground straight up to the top of the wall is 13 feet Will the ladder reach the top of the wall?

STATISTICS AND PROBABILITY (SP)			
Summarize and describe distributions.			
4.SP.1 MWOTL	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data was gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data was gathered.	CC.6.SP.5 CCR.SP.D	Determining a grade point average Ex. a. & b. Given data on heights, weights, temperatures, speeds, etc. Describe the data's attributes, units of measurement, tool used to measure, and number of observations Ex. c. An auto manufacturer wants their line of cars to have a median gas mileage of 25 miles per gallon or higher. The gas mileage for their five models are 23, 25, 26, 29, and 19. Do their cars meet their goal? Ex. d. The number of people who attended an art conference for five days was 42, 27, 35, 39, and 96. Describe the effect of the outlier on the mean and median.
Investigate chance processes and develop, use, and evaluate probability models.			
4.SP.2 MWOTL	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	CC.7.SP.5 CCR.SP.D	Deciding whether or not to carry an umbrela Ex. The weather forecast states that the probability it will rain tomorrow is 0.30. Is it unlikely, neither likely nor unlikely, or likely that it will rain tomorrow?

4.SP.3 MWOTL	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	CC.7.SP.6 CCR.SP.D	Ex. When rolling a number cube 600 times, predicting that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
4.SP.4	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	CC.7.SP.7 CCR.SP.D	Tossing a coin Ex. Selecting a student at random from a class, finding the probability that Jane will be selected and the probability that another female student will be selected. Ex. Finding the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

4.SP.5	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. c. Design and use a simulation to generate frequencies for compound events.	CC.7.SP.8 CCR.SP.D	Rolling dice or tossing a coin For an event described in everyday language (e.g., "rolling double sixes"), identifying the outcomes in the sample space which compose the event Use random digits as a simulation tool to approximate the answer to the question Ex. If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
Investigate patterns of association in bivariate data.			
4.SP.6 MWOTL	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	CC.8.SP.1 CCR.SP.D	Plot and analyze related student attributes (i.e., bed times and wake up times, workdays per week and times eating out per week, number of children, number of hours of sleep per night)
4.SP.7 MWOTL	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear relationship, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	CC.8.SP.2 CCR.SP.D	Plot and analyze related student attributes (i.e., bed times and wake up times, workdays per week and times eating out per week, number of children, number of hours of sleep per night)

4.SP.8 MWOTL	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	CC.8.SP.3 CCR.SP.D	Plot and analyze related student attributes (i.e., bed times and wake up times, workdays per week and times eating out per week, number of children, number of hours of sleep per night) Ex. In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
4.SP.9 MWOTL	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	CC.8.SP.4 ESS01.03.06 CCR.SP.D	Ex. Collect data from students on whether or not they drink coffee and whether or not they have trouble sleeping at night. Is there evidence that those who drink coffee also tend to have trouble sleeping at night?

NRS Level 5 Overview

Number and Quantity (N)

The Real Number System (RN)

- Apply and extend previous understandings of operations with fractions to add, subtr multiply, and divide rational numbers.
- Know that there are numbers that are not rational, and approximate them by rationa numbers.
- Use properties of rational and irrational numbers

Quantities (Q)

- Reason quantitatively and use units to solve problems

Algebra (A)

Seeing Structure in Expressions (SSE)

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Reasoning with Equations and Inequalities (REI)

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Functions (F)

Interpreting Functions (IF)

- Analyze functions using different representations

Building Functions (BF)

- Build a function that models a relationship between two quantities

NRS Level 5 Overview (continued)

Geometry (G)

Congruence (CO)

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry (SRT)

- Understand similarity in terms of similarity transformations
- Understand and apply the Pythagorean Theorem.
- Prove theorems involving similarity

Circles (C)

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors

Expressing Geometric Properties with Equations (GPE)

- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension (GMD)

- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
- Explain volume formulas and use them to solve problems

Modeling with Geometry (MG)

- Apply geometric concepts in modeling situations

NRS Level 5 Overview (continued)

Statistics and Probability (S)

Interpreting Categorical and Quantitative Data (ID)

- Use random sampling to draw inferences about a population
- Draw informal comparative inferences about two populations
- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions (IC)

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies of circles

Using Probability to Make Decisions (MD)

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

★ Modeling
\+ Provides foundations for advanced college courses

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS LEVEL 5 – High Intermediate Adult Basic Education (Grade Levels 7-8)
NRS Level 6 - Adult Secondary Education (Grade Levels 9-12)

Difference between Adult Education and High School Education
The literacy through low intermediate standards presented in this document provide for a fairly sequential progression of mathematics instruction and learning. Low and high ASE level standards together prepare adult education students for the rigors of high school level mathematics.

The low and high ASE level standards are grouped into six conceptual categories, each of which is further divided into domain groupings. Adult educators should be aware that specific order, as with all standards, should be left up to individual teachers and programs, not necessarily having to complete all of Level 5 before moving on to Level 6.

College and Career Ready
Low and high ASE standards specify the mathematics that all students should study in order to be college and career ready[20]. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics are indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

It should also be noted that the goal is to be college ***and*** career ready, and levels 5 and 6 represent the entirety of this goal. Level 5 is not exclusively for HSE/Career preparedness, and Level 6 is not exclusively college ready.

Notes on Courses and Transitions
Currently, most high schools use a traditional sequence of Algebra I, Geometry, and Algebra II. This has been slowly changing over the years, as some schools see the advantage of using an Algebra I, Algebra II, and then Geometry sequence. These standards do not mandate a sequence of courses.

A major transition for adults is the transition from adult education to post-secondary education for college and careers. The evidence concerning college and career

[20] Massachusetts Curriculum Framework for Mathematics, 2011, www.doe.mass.edu/frameworks/current

readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics. Important standards for college and career readiness are distributed across levels and courses. Some of the highest priority content for college and career readiness comes from NRS level 4 (5.0-8.9). This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. It is important to note as well that information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Explanation of Conceptual Categories

The Adult Secondary Education standards are listed in conceptual categories:

I. Number and Quantity
II. Algebra
III. Functions
IV. Modeling
V. Geometry
VI. Statistics and Probability

These six conceptual categories portray a coherent view of Adult Secondary Education based on the common core high school standards; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Conceptual Category I - Number and Quantity

Numbers and Number Systems - During NRS Levels 1-4, adult education students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. Also, during levels 1-4, fractions are augmented by negative fractions to form the rational numbers. And during NRS Level 4 students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. ASE level students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties, and their new meanings are consistent with their previous meanings.

Quantities - In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. ASE students encounter a wider variety of units in modeling (e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages). They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification.

Conceptual Category II – Algebra

Expressions - An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities - An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = \left(\frac{b1+b2}{2}\right)h$, can be solved for *h* using the same deductive process.
Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling - Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Conceptual Category III – Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, *v*; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is *T*.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

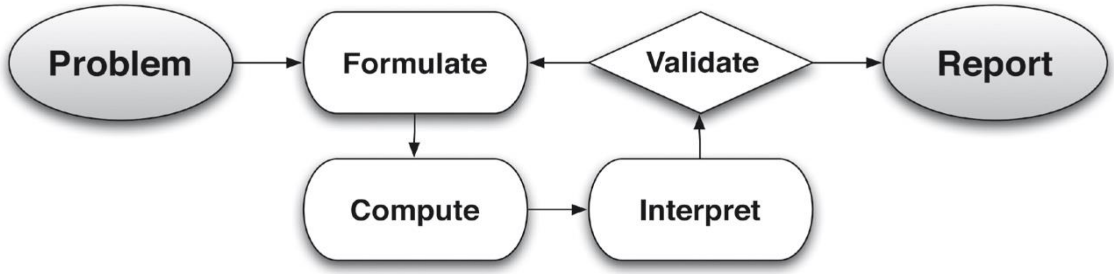
Connections to Expressions, Equations, Modeling, and Coordinates - Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Conceptual Category IV – Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the above diagram. It involves the following:

1. Identifying variables in the situation and selecting those that represent essential features.
2. Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables.
3. Analyzing and performing operations on these relationships to draw conclusions.
4. Interpreting the results of the mathematics in terms of the original situation.
5. Validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
6. Reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the ASE standards indicated by a star symbol (★).

Conceptual Category V – Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations - The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

<u>Conceptual Category VI – Statistics and Probability★</u>

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed, as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling - Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical understanding and expertise.

Mathematical Practices

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★ Modeling
\+ Provides foundations for advanced college courses

NRS Level 5 – High Intermediate Basic Education
(Grade Levels 7-8)

Standard Number	MATH STANDARD	Reference	Example task, problem, or activity
NUMBER AND QUANTITY (N)			
The Real Number System (RN)			
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.			
5.N.RN.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	CC.7.NS.1 CCR.NS.D	Ex. Aakash, Bao Ying, Chris and Donna all live on the same street as their office building, which runs from east to west. Aakash lives 5 $\frac{1}{2}$ blocks to the west. Bao Ying lives 4 $\frac{1}{4}$ blocks to the east. Chris lives 2 $\frac{3}{4}$ blocks to the west. Donna lives 6 $\frac{1}{2}$ blocks to the east. Represent the relative position of the houses on a number line, with the office at zero, points to the west represented by negative numbers, and points to the east represented by positive numbers. How far is each house from the office and from each other?

	(NOTE: This standard was partially addressed in NRS 4. *In NRS 5 the focus is on interpreting rational number sums by identifying the numbers as quantities in real-world situations, using absolute value to develop an efficient algorithm for subtraction problems that involve negative rational numbers and applying the algorithms in real-world problems.*)		
5.N.RN.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then $-(\frac{p}{q}) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (NOTE: This standard was partially addressed in NRS 4. *In NRS 5 the focus is on applying the algorithms in real-world problems.)*	CC.7.NS.2 ESS01.03.02 CCR.NS.D	Ex. A water well drilling rig has dug to a height of –60 feet after one full day of continuous use. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours? If the rig has been running constantly and is currently at a height of –143.6 feet, for how long has the rig been running?

Know that there are numbers that are not rational, and approximate them by rational numbers.			
5.N.RN.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) (NOTE: This standard was partially addressed in NRS 4.)	CC.7.NS.3 ESS01.03.02 CCR.NS.D	Ex. Three departments at Sunview Supermarket collected the most canned goods for a local fundraiser, and so they won a $600 prize to share among them. The produce department collected 3,760 cans, the meat department collected 2,301, and the dairy department collected 1,855. How should they divide the money so that each department gets the same fraction of the prize money as the fraction of the canned goods that they collected?
5.N.RN.4	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0's or eventually repeat. Know that other numbers are called irrational. (NOTE: This standard was partially addressed in NRS 4.)	CC.8.NS.1	
5.N.RN.5	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2 (NOTE: This standard was partially addressed in NRS 4.)	CC.8.NS.2 ESS01.03.06 CCR.NS.D	Ex. By truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Use properties of rational and irrational numbers.			
5.N.RN.6	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	CC.N.RN.3	
QUANTITIES★ (Q)			
Reason quantitatively and use units to solve problems.			
5.N.Q.1 MWOTL	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	CC.N.Q.1 CCR.N.Q.E	Conversing about information contained in newspapers and magazines
5.N.Q.2	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	CC.7.RP.3 ESS01.03.02 CCR.RP.D	Determining the savings when buying something 50% off Determining the difference in tip at 15% or 20% of bill
ALGEBRA (A)			
Seeing Structure in Expressions (SSE)			
Interpret the structure of expressions.			
5.A.SSE.1 MWOTL a only	Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.	CC.A.SSE.1 CCR.A.SSE.E	Entering an expression/formula in a spreadsheet Keeping personal finance records

CREATING EQUATIONS (CED)			
Create equations that describe numbers or relationships.			
5.A.CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* (NOTE: This standard will also be addressed in NRS 6.)	CC.A.CED.1 CCR.A.CED.E	Plot and analyze related student attributes (i.e., examples of quadratic distributions needed)
5.A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (NOTE: This standard will be fully addressed in NRS 6.)	CC.A.CED.2 CCR.A.CED.E	Scatter plot and analyze related student attributes
5.A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. (NOTE: This standard will be fully addressed in NRS 6.)	CC.A.CED.3 CCR.A.CED.E	Representing inequalities describing nutritional and cost constraints on combinations of different foods
5.A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (NOTE: This standard will be fully addressed in NRS 6.)	CC.A.CED.4 CCR.A.CED.E	Rearranging Ohm's law V = IR to highlight resistance R

REASONING WITH EQUATIONS AND INEQUALITIES (REI)			
Understand solving equations as a process of reasoning and explain the reasoning.			
5.A.REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	CC.A.REI.1 CCR.A.REI.E	

Solve equations and inequalities in one variable.			
5.A.REI.2 MWOTL	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	CC.A.REI.3 CCR.A.REI.E	Finding an unknown amount of a check given current balance Solving straight-line distance equations such as trains moving in opposite directions
Solve systems of equations.			
5.A.REI.3	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	CC.A.REI.5	
5.A.REI.4	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	CC.A.REI.6 CCR.A.REI.E	Comparing cost of production to profit for a bake sale Use supply/demand lines to find balance of a market
Represent and solve equations and inequalities graphically.			
5.A.REI.5	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	CC.A.REI.10 CCR.A.REI.E	
FUNCTIONS (F)			
INTERPRETING FUNCTIONS (IF)			
Analyze functions using different representations.			
5.F.IF.1	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	CC.F.IF.7 CCR.F.IF.E	Scatter plot and analyze related student attributes (i.e., examples of exponential and polynomial distributions needed)

BUILDING FUNCTIONS (BF)			
Build a function that models a relationship between two quantities.			
5.F.BF.1 MWOTL	Write a function that describes a relationship between two quantities. ★	CC.F.BF.1 CCR.F.BF.E	Figuring the effect on mortgage payments of a change in interest rates Calculate total costs based on fixed costs and variable costs
5.F.BF.2	Determine an explicit expression, a recursive process, or steps for calculation from a context.	CC.F.BF.2	
GEOMETRY (G)			
CONGRUENCE (CO)			
Experiment with transformations in the plane.			
5.G.CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	CC.G.CO.1 CCR.G.CO.E	
5.G.CO.2	Represent transformations in the plane using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	CC.G.CO.2	Applications in creating art
5.G.CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	CC.G.CO.3	Graphic design
5.G.CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	CC.G.CO.4	Graphic design software development

5.G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	CC.G.CO.5	Artistic applications
Understand congruence in terms of rigid motions.			
5.G.CO.6	Use geometric descriptions of rigid motions to transform figures and predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	CC.G.CO.6	Graphic design Software development (such as CAD/CAM, CNC)
5.G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	CC.G.CO.7	
5.G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	CC.G.CO.8	
Prove geometric theorems.			
5.G.CO.9	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	CC.G.CO.9	
5.G.CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	CC.G.CO.10	

5.G.CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	CC.G.CO.11	
Make geometric constructions.			
5.G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	CC.G.CO.12	
5.G.CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	CC.G.CO.13	
Similarity, Right Triangles, and Trigonometry (SRT)			
Understand similarity in terms of similarity transformations.			
5.G.SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	CC.G.SRT.1	Create an enlargement
5.G.SRT.2	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. (NOTE: This standard was partially addressed in NRS 4.)	CC.8.G.5 CCR.G.D	Ex. Arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.

5.G.SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	CC.G.SRT.3	Verify a scale drawing
Understand and apply the Pythagorean Theorem.			
5.G.SRT.4	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	CC.8.G.8 CCR.G.D	Map applications
Prove theorems involving similarity.			
5.G.SRT.5	Prove theorems about triangles. Theorems include*: a line parallel to one side of a triangle divides the other two proportionally; and conversely, the Pythagorean Theorem proved using triangle similarity.*	CC.G.SRT.4	
5.G.SRT.6 MWOTL	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	CC.G.SRT.5 CCR.G.SRT.E	Determine the height of an object using shadows
CIRCLES (C)			
Understand and apply theorems about circles.			
5.G.C.1	Prove that all circles are similar.	CC.G.C.1	
5.G.C.2	Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	CC.G.C.2	
5.G.C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	CC.G.C.3	
5.G.C.4	(+) Construct a tangent line from a point outside a given circle to the circle.	CC.G.C.4	Plotting the customer line for a carousel in designing a fair floor plan.
Find arc lengths and areas of sectors of circles.			
5.G.C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	CC.G.C.5	

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (GPE)			
Use coordinates to prove simple geometric theorems algebraically.			
5.G.GPE.1	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	CC.G.GPE.5	
5.G.GPE.2	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	CC.G.GPE.6	Segment a multi-day trip into daily sections
5.G.GPE.3	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).★	CC.G.GPE.7	Map applications: distance travelled on a multi-stop journey
GEOMETRIC MEASUREMENT AND DIMENSION (GMD)			
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.			
5.G.GMD.1	Know the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	CC.8.G.9	Determine capacity of a silo, industrial bin, waffle cone
Explain volume formulas and use them to solve problems.			
5.G.GMD.2	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	CC.G.GMD.1	
5.G.GMD.3 MWOTL	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	CC.G.GMD.3 CCR.G.GMD.E	Determine capacity of a silo, industrial bin, waffle cone, hot air balloon
MODELING WITH GEOMETRY (MG)			
Apply geometric concepts in modeling situations.			
5.G.MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	CC.G.MG.1	Artistic applications, graphic design, engineering design

STATISTICS AND PROBABILITY (S)			
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA (ID)			
Use random sampling to draw inferences about a population.			
5.S.ID.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	CC.7.SP.1 CCR.SP.D	Conducting a political survey
5.S.ID.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	CC.7.SP.2 CCR.SP.D	Estimating the mean word length in a book by randomly sampling words from the book. Gauge how far off the estimate or prediction might be.
Draw informal comparative inferences about two populations.			
5.S.ID.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	CC.7.SP.3 CCR.SP.D	Comparing personal financial choices to data from an ideal spender's purchase log Ex. The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

5.S.ID.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	CC.7.SP.4 CCR.SP.D	Ex. Deciding whether the words in a chapter of NRS Level 1 science book are generally longer than the words in a chapter of NRS level 4 science book
Summarize, represent, and interpret data on a single count or measurement variable.			
5.S.ID.5 MWOTL	Represent data with plots on the real number line (dot plots, histograms, and box plots).	CC.S.ID.1 CCR.S.ID.E	Taking evidence based political action to institute changes in the community, track student progress, crop growth, savings rate
5.S.ID.6	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	CC.S.ID.2	Understanding there are no “true” measures for statistical quantities (i.e., home values, expected wages, expected prices, expected profits or losses)
5.S.ID.7 MWOTL	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	CC.S.ID.3 CCR.S.ID.E	Understanding there are no “true” measures for statistical quantities (i.e., home values, expected wages, expected prices, expected profits or losses)
5.S.ID.8	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	CC.S.ID.4 ESS04.07.01 ESS04.07.02	Tracking one’s daily expenses personal financial choices to data from an ideal spender’s purchase log Child’s height/weight compared to the norm: understanding a blood test report

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS (IC)			
Understand and evaluate random processes underlying statistical experiments.			
5.S.IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	CC.S.IC.1	Taking evidence based political action to institute changes in the community Interpret internet news articles
5.S.IC.2	Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	CC.S.IC.2	Analyze accident occurrences at given intersections Ex. A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.			
5.S.IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	CC.S.IC.3	Interpret internet information
5.S.IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	CC.S.IC.4	Population weight, opinion survey
5.S.IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	CC.S.IC.5	Vaccine testing
5.S.IC.6	Evaluate reports based on data.	CC.S.IC.6	Bureau of Labor Statistics

USING PROBABILITY TO MAKE DECISIONS (MD)			
Calculate expected values and use them to solve problems.			
5.S.MD.1	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	CC.S.MD.1	Representing findings from data gathering in a manufacturing, medical or business setting
5.S.MD.2	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	CC.S.MD.2	Gathering data in the workplace and sorting it by criteria Card games
5.S.MD.3	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.	CC.S.MD.3	Comparing gathered work-related data by preparation of appropriate bar or line graphs Ex. Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

5.S.MD.4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?	CC.S.MD.4	Comparing gathered work-related data by preparation of appropriate bar or line graphs Ex. Find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?
Use probability to evaluate outcomes of decisions.			
5.S.MD.5	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values.	CC.S.MD.5	Interpreting the odds of contracting breast cancer or being in an airplane accident Ex. a. Interpreting the odds of winning a contest based on the number of entries Ex. b. Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
5.S.MD.6	(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	CC.S.MD.6	Determining chances of drawing “short stick”
5.S.MD.7	(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	CC.S.MD.7	Determining the outcomes of decisions based on known variables

NRS Level 6 Overview

(Please see *Overview Explanation of Mathematics NRS Levels 5 & 6* for further information.)

Number and Quantity (N)

The Real Number System (RN)

- Extend the properties of exponents to rational exponents

Quantities (Q)

- Reason quantitatively and use units to solve problems

The Complex Number System (CN)

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities (VM)

- Represent and model with vector quantities
- Perform operations on vectors
- Perform operations on matrices and use matrices in applications

Algebra (A)

Seeing Structure in Expressions (SSE)

- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions (APR)

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations (CED)

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities (REI)

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve systems of equations
- Represent and solve equations and inequalities graphically

NRS Level 6 Overview (continued)

Functions (F)

Interpreting Functions (IF)

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions (BF)

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models (LE)

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions (TF)

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Geometry (G)

Similarity, Right Triangles, and Trigonometry (SRT)

- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Expressing Geometric Properties with Equations (GPE)

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension (GMD)

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry (MG)

- Apply geometric concepts in modeling situations

NRS Level 6 Overview (continued)

Statistics and Probability (S)

Conditional Probability and the Rules of Probability (CP)

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

★	Modeling
+	Provides foundations for advanced college courses

NRS Level 6 – Adult Secondary Education
(Grade Levels 9-12)

Standard Number	MATH STANDARD	Reference	Example task, problem, or activity
NUMBER AND QUANTITY (N)			
QUANTITIES★ (Q)			
Reason quantitatively and use units to solve problems.			
6.N.Q.1	Define appropriate quantities for the purpose of descriptive modeling.	CC.N.Q.2	Ex. A small company wants to give raises to their 5 employees. They have $10,000 available to distribute. Imagine you are in charge of deciding how the raises should be determined. What are some variables you should consider? Describe mathematically different methods to distribute the raises. What information do you need to compute the raises for each employee? Make up the information you need to compute specific raises for 2 different methods and apply them to the situation. Compute the specific dollar mount each employee receives as a raise. Choose one of the methods that you think is most fair and construct an argument that supports your decision. (illustrativemathematics.org)

6.N.Q.2 MWOTL	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★	CC.N.Q.3 CCR.N.Q.E	Ex. Quincy is a tour guide at a museum of science and history. During a tour of the museum, he tells some visitors about a fossilized dinosaur bone that is on display in the museum. He says, "Twenty years ago, a group of paleontologists donated this dinosaur bone to our museum. At the time, they told us that they had estimated the age of the bone to be approximately 90 million years. So now, the bone is about 90 million and 20 years old." Evaluate the validity of Quincy's statement. (illustrativemathematics.org)
THE REAL NUMBER SYSTEM (RN)			
Extend the properties of exponents to rational exponents.			
6.N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	CC.N.RN.1	For example, we define $5^{1/3}$ to be the cube root of 5 because we want $\left(5^{1/3}\right)^3 = 5^{(1/3)^3}$to hold, so $\left(5^{1/3}\right)^3$must equal 5.
6.N.RN.2 MWOTL	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	CC.N.RN.2 CCR.N.RN.E	
THE COMPLEX NUMBER SYSTEM (CN)			
Extend the properties of exponents to rational exponents.			
Perform arithmetic operations with complex numbers.			
6.N.CN.1	Know there is a complex number i such that i^2 = –1, and every complex number has the form *a* + *bi* with *a* and *b* real.	CC.N.CN.1	
6.N.CN.2	Use the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	CC.N.CN.2	

6.N.CN.3	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	CC.N.CN.3	
Represent complex numbers and their operations on the complex plane.			
6.N.CN.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	CC.N.CN.4	
6.N.CN.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $\left(-1+\sqrt{3i}\right)^3 = 8$ *because* $\left(-1+\sqrt{3i}\right)$ *has modulus* 2 *and argument* 120°.	CC.N.CN.5	
6.N.CN.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	CC.N.CN.6	
Use complex numbers in polynomial identities and equations.			
6.N.CN.7	Solve quadratic equations with real coefficients that have complex solutions.	CC.N.CN.7	
6.N.CN.8	(+) Extend polynomial identities to the complex numbers. *For example, rewrite* x^2+4 *as* $(x+2i)(x-2i)$.	CC.N.CN.8	
6.N.CN.9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	CC.N.CN.9	

VECTOR AND MATRIX QUANTITIES (VM)			
Represent and model with vector quantities.			
6.N.VM.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v).	CC.N.VM.1	Studying vector forces on an object (e.g., in physics)
6.N.VM.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	CC.N.VM.2	Understanding how weather affects flight times and paths
6.N.VM.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.	CC.N.VM.3	Studying vector forces on an object (e.g., in physics)
Perform operations on vectors.			
6.N.VM.4	(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	CC.N.VM.4	Studying vector forces on an object (e.g., in physics)

6.N.VM.5	(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|v. Compute the direction of c**v** knowing that when |c|v ≠ 0, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).	CC.N.VM.5	Vector motion diagrams
Perform operations on matrices and use matrices in applications.			
6.N.VM.6	(+) Use matrices to represent and manipulate data	CC.N.VM.6	Represent payoffs or incidence relationships in a network
6.N.VM.7	(+) Multiply matrices by scalars to produce new matrices.	CC.N.VM.7	When all of the payoffs in a game are doubled
6.N.VM.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.	CC.N.VM.8	
6.N.VM.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	CC.N.VM.9	
6.N.VM.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	CC.N.VM.10	
6.N.VM.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	CC.N.VM.11	Vector force diagrams

6.N.VM.12	(+) Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	CC.N.VM.12	
ALGEBRA (A)			
SEEING STRUCTURE IN EXPRESSIONS (SSE)			
Write expressions in equivalent forms to solve problems.			
6.A.SSE.1 MWOTL	Use the structure of an expression to identify ways to rewrite it.	CC.A.SEE.2 CCR.A.SSE.E	Ex. See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
6.A.SSE.2 MWOTL c only	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ a. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. b. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. c. Factor a quadratic expression to reveal the zeros of the function it defines.	CC.A.SSE.3 CCR.A.SSE.E ESS04.08.01	Converting annual interest to monthly interest Understanding exponential growth of bacteria or virus such as HIV Converting annual interest to monthly interest
6.A.SSE.3	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.★	CC.A.SSE.4	Calculate mortgage payments

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS (APR)			
Perform arithmetic operations on polynomials.			
6.A.APR.1 MWOTL	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	CC.A.APR.1 ESS01.03.04 ESS01.03.05 CCR.A.APR.E	
Understand the relationship between zeros and factors of polynomials.			
6.A.APR.2	Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x* – *a* is *p*(*a*), so *p*(*a*) = 0 if and only if (*x* – *a*) is a factor of *p*(*x*).	CC.A.APR.2	
6.A.APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	CC.A.APR.3	Maximize area of garden with constraints, analyze path of a roller coaster
Use polynomial identities to solve problems.			
6.A.APR.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	CC.A.APR.4	
6.A.APR.5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.	CC.A.APR.5	

Rewrite rational expressions.			
6.A.APR.6 MWOTL	Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x)* + *r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	CC.A.APR.6 CCR.A.APR.E	
6.A.APR.7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	CC.A.APR.7	
CREATING EQUATIONS (CED)			
Create equations that describe numbers or relationships.			
6.A.CED.1 MWOTL	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ★ (NOTE: This standard was partially addressed in NRS 5.)	CC.A.CED.1 CCR.A.CED.E	Plot and analyze related student attributes (i.e., examples of quadratic distributions needed) Managing costs, tracking a projectile; calculate expected salary subject to an annual compound increase (such as for state employees)
6.A.CED.2 MWOTL	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ (NOTE: This standard was partially addressed in NRS 5.)	CC.A.CED.2 CCR.A.CED.E	Scatter plot and analyze related student attributes (i.e., examples of exponential and polynomial distributions needed) Distances and times of marathon runners

6.A.CED.3 MWOTL	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. ★ (NOTE: This standard was partially addressed in NRS 5.)	CC.A.CED.3 CCR.A.CED.E	Representing inequalities describing nutritional and cost constraints on combinations of different foods
6.A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (NOTE: This standard was partially addressed in NRS 5.)	CC.A.CED.4 CCR.A.CED.E	Ex. Rearranging Ohm's law V = IR to highlight resistance R
REASONING WITH EQUATIONS AND INEQUALITIES (**REI**)			
Understand solving equations as a process of reasoning and explain the reasoning.			
6.A.REI.1	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	CC.A.REI.2 NETS•S 4d	Distance equations
6.A.REI.2 MWOTL	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection $(e.g., for\ x^2 = 49)$, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b.*	CC.A.REI.4 CCR.A.REI.E	Plot and analyze related student attributes (i.e., examples of quadratic distributions needed)

Solve systems of equations.			
6.A.REI.3	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle $x^2 + y^2 = 3$.	CC.A.REI.7	Bodies in motion
6.A.REI.4	(+) Represent a system of linear equations as a single matrix equation in a vector variable.	CC.A.REI.8	
6.A.REI.5	(+) Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	CC.A.REI.9	
Represent and solve equations and inequalities graphically.			
6.A.REI.6	Explain why the x-coordinates of the points where the graphs of the equations *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation *f(x)* = *g(x)*; find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	CC.A.REI.11 ESS04.08.02 ESS04.08.03	Bodies in motion
6.A.REI.7	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	CC.A.REI.12	Costs of different resources: find maximum value

FUNCTIONS (F)			
INTERPRETING FUNCTIONS (IF)			
Understand the concept of a function and use function notation.			
6.F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	CC.F.IF.1 CCR.F.IF.E	
6.F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	CC.F.IF.2 ESS04.08.03 CCR.F.IF.E	Height of a projectile, motion equations, revenue/cost equations, production equations
6.F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	CC.F.IF.3 ESS04.08.03	Ex. The Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.
Interpret functions that arise in applications in terms of the context.			
6.F.IF.4 MWOTL	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	CC.F.IF.4 ESS01.03.06 CCR.F.IF.E	Describing change in workplace output relative to contributing factors (i.e., number of employees, shift time, time of year)
6.F.IF.5	Graph linear and quadratic functions and show intercepts, maxima, and minima.	CC.F.IF.7a	

6.F.IF.6 MWOTL	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★	CC.F.IF.5 CCR.F.IF.E	Reading a chart or graph in a health pamphlet Ex. If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6.F.IF.7	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	CC.F.IF.6 CCR.F.IF.E	Conversing about information in newspapers and magazines
Analyze functions using different representations.			
6.F.IF.8 MWOTL	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	CC.F.IF.7 ESS01.03.06	Scatter plot and analyze related student attributes (i.e., examples of exponential and polynomial distributions needed) Examine decibel scale and Richter scale

6.F.IF.9 MWOTL b only	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	CC.F.IF.8 CCR.F.IF.E	Tracking monthly savings plan without interest Plot and analyze related student attributes (i.e., bed times and wake up times, workdays per week and times eating out per week, number of children, number of hours of sleep per night) Ex. b. For example, identify percent rate of change in functions such as $\mathrm{y} = (1.02)^{\mathrm{t}}$, $\mathrm{y} = (0.97)^{\mathrm{t}}$, $\mathrm{y} = (1.01)^{12\mathrm{t}}$, $\mathrm{y} = (1.2)^{\mathrm{t}/10}$, and classify them as representing exponential growth or decay.
6.F.IF.10 MWOTL	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	CC.F.IF.9 CCR.F.IF.E	Reading a graph in an ad or poster Ex. Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
BUILDING FUNCTIONS (BF)			
Build a function that models a relationship between two quantities.			
6.F.BF.1	Combine standard function types using arithmetic operations. a. (+) Compose functions.	CC.F.BF.1	Building a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relating these functions to the model Ex. If T(*y*) is the temperature in the atmosphere as a function of height, and *h(t)* is the height of a weather balloon as a function of time, then T(*h*(*t*)) is the temperature at the location of the weather balloon as a function of time.

6.F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	CC.F.BF.2	Keeping personal finance records
Build new functions from existing functions.			
6.F.BF.3	Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	CC.F.BF.3 NETS•S 4d	
6.F.BF.4	Find inverse functions. a. Solve an equation of the form *f(x)* = *c* for a simple function *f* that has an inverse and write an expression for the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	CC.F.BF.4	Entering an expression in a spreadsheet Keeping personal finance records Ex. a. $\mathrm{f(x) = 2x^3}$ or $\mathrm{f(x) = (x+1)/(x-1)}$ for $\mathrm{x \neq 1}$
6.F.BF.5	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	CC.F.BF.5	Can be used in any exponential context: growth or decay Decibel and Richter scales

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS★ (LE)			
Construct and compare linear, quadratic, and exponential models and solve problems.			
6.F.LE.1 MWOTL	Distinguish between situations that can be modeled with linear functions and with exponential functions. ★ a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	CC.F.LE.1 CCR.F.LE.E	Understanding the importance of investing early to take advantage of compounded returns (i.e., college or retirement)
6.F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).	CC.F.LE.2	Setting realistic investment goals (i.e., college or retirement) Comparing straight savings to investing with compound interest.
6.F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	CC.F.LE.3	Discussing with a financial planner about retirement investment plans offered at work Analyzing growth of bacterial culture
6.F.LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.	CC.F.LE.4	Can be used in any exponential context: growth or decay; decibel and Richter scales
Interpret expressions for functions in terms of the situation they model.			
6.F.LE.5 MWOTL	Interpret the parameters in a linear or exponential function in terms of a context. ★	CC.F.LE.5 CCR.F.LE.E	Setting realistic investment goals (i.e., college or retirement)
TRIGONOMETRIC FUNCTIONS (TF)			
Extend the domain of trigonometric functions using the unit circle.			
6.F.TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	CC.F.TF.1	

6.F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	CC.F.TF.2	
6.F.TF.3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3, \pi/4, and\ \pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x, and\ 2\pi - x$ in terms of their values for *x*, where *x* is any real number.	CC.F.TF.3	
6.F.TF.4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	CC.F.TF.4	
Model periodic phenomena with trigonometric functions.			
6.F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	CC.F.TF.5	Creating a trigonometric function to model heating costs throughout the year
6.F.TF.6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	CC.F.TF.6	
6.F.TF.7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★	CC.F.TF.7	Navigation, surveying
Prove and apply trigonometric identities.			
6.F.TF.8	Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $\sin(\theta), \cos(\theta), or \tan(\theta)$ given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	CC.F.TF.8	
6.F.TF.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	CC.F.TF.9	
GEOMETRY (G)			

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY (SRT)			
Define trigonometric ratios and solve problems involving right triangles.			
6.G.SRT.1	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	CC.G.SRT.6	Building and designing structures
6.G.SRT.2	Explain and use the relationship between the sine and cosine of complementary angles.	CC.G.SRT.7	Designing products
6.G.SRT.3	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	CC.G.SRT.8	Designing products Compute distances, determine angles of elevation/declination, quilt design
Apply trigonometry to general triangles.			
6.G.SRT.4	(+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	CC.G.SRT.9	Designing products
6.G.SRT.5	(+) Prove the Laws of Sines and Cosines and use them to solve problems.	CC.G.SRT.10	Designing products
6.G.SRT.6	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	CC.G.SRT.11	Designing products Surveying, force vector diagram
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (GPE)			
Translate between the geometric description and the equation for a conic section.			
6.G.GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	CC.G.GPE.1	Using CAD/CAM software to design a product
6.G.GPE.2	Derive the equation of a parabola given a focus and directrix.	CC.G.GPE.2	Reading scientific diagrams
6.G.GPE.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	CC.G.GPE.3	Using CAD/CAM software to design a product
Use coordinates to prove simple geometric theorems algebraically.			

6.G.GPE.4	Use coordinates to prove simple geometric theorems algebraically.	CC.G.GPE.4	Proving or disproving that a figure defined by four given points in the coordinate plane is a rectangle; proving or disproving that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)
GEOMETRIC MEASUREMENT AND DIMENSION (GMD)			
Explain volume formulas and use them to solve problems.			
6.G.GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	CC.G.GMD.1	Building and measuring structures and objects
6.G.GMD.2	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	CC.G.GMD.2	Building and measuring structures and objects
Visualize relationships between two-dimensional and three-dimensional objects.			
6.G.GMD.3	Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	CC.G.GMD.4	Building and measuring structures and objects Interpreting medical scans Understanding 3-D printing
MODELING WITH GEOMETRY (MG)			
Apply geometric concepts in modeling situations.			
6.G.MG.1	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★	CC.G.MG.3	Building and measuring structures and objects
6.G.MG.2	Apply concepts of density based on area and volume in modeling situations.★	CC.G.MG.2 CCR.G.MG.E	Persons per square mile, BTUs per cubic foot

STATISTICS AND PROBABILITY (SP)			
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA (ID)			
Summarize, represent, and interpret data on two categorical and quantitative variables.			
6.S.ID.1 MWOTL	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.*	CC.S.ID.5 CCR.S.ID.E	Looking at reports on stock market to see if they reflect the trends represented
6.S.ID.2	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	CC.S.ID.6 ESS04.08.01 ESS04.08.02	Tracking monthly savings plan without interest Plot and analyze related student attributes (i.e., bed times, wake up times, workdays per week, times eating out per week, number of children, hours of sleep per night); examples of quadratic distributions needed and exponential and polynomial distributions
Interpret linear models.			
6.S.ID.3	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	CC.S.ID.7 CCR.S.ID.E	Demand curve for a product. Supply curve for a product.
6.S.ID.4	Compute (using technology) and interpret the correlation coefficient of a linear fit.	CC.S.ID.8 ESS04.08.03	
6.S.ID.5	Distinguish between correlation and causation.	CC.S.ID.9 CCR.S.ID.E	Taking political action to institute changes in the community Diagnosing an illness from symptoms

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY (CP)			
Understand independence and conditional probability and use them to interpret data.			
6.S.CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	CC.S.CP.1	Tossing a coin or rolling a die Possible hands in a card game Possible outcomes of a restaurant contest
6.S.CP.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	CC.S.CP.2	Tossing a coin or rolling a die Choosing matching clothes combinations (e.g., of shirts and pants)
6.S.CP.3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	CC.S.CP.3	Tossing a coin or rolling a die Choosing from a bag of candies to get different colors Matching socks from a pile
6.S.CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your class on their favorite subject among math, science, and English.*	CC.S.CP.4 ESS01.03.06	Estimating the probability that a randomly selected student from class will favor science. Do the same for other subjects and compare the results
6.S.CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	CC.S.CP.5	Comparing the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer

Use the rules of probability to compute probabilities of compound events in a uniform probability model.			
6.S.CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	CC.S.CP.6	Tossing a coin or rolling a die Seating members of a panel
6.S.CP.7	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	CC.S.CP.7	Tossing a coin or rolling a die
6.S.CP.8	(+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	CC.S.CP.8	Tossing a coin or rolling a die Calculate probability of winning in card games
6.S.CP.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems.	CC.S.CP.9	Tossing a coin or rolling a die Compute varieties of flower arrangements in a corsage

Illinois ABE/ASE Content Standards
Math Glossary

A

AA similarity: Angle-angle similarity. When two triangles have corresponding angles that are congruent, the triangles are similar.

Absolute value: a) A nonnegative number equal in numerical value to a given real number; b) The distance from a number to zero on the number line.

Acute angle: Measures less than 90°.

Acute triangle: Has three acute angles.

Addition and subtraction within 5, 10, 20, 100, or 1000: Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses: Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and -3/4 are additive inverses of one another because $\frac{3}{4} + \left(-\frac{3}{4}\right) = \left(-\frac{3}{4}\right) + \left(\frac{3}{4}\right) = 0$.

Adjacent angles: Angles that have a common vertex and a common ray.

Algebraic expressions: Like numerical expressions, except that both letters and numbers can be used in such expressions.

Algorithm: A finite set of steps for completing a procedure (e.g., long division).

Alternate exterior angles: A pair of congruent angles formed by two parallel lines cut by a transversal, located outside the parallel lines and on opposite sides of the transversal.

Alternate interior angles: A pair of congruent angles formed by two parallel lines cut by a transversal, located inside the parallel lines and on opposite sides of the transversal.

Analog: Data represented by continuous variables (e.g., a clock with hour, minute, and second hands).

Analytic geometry: The branch of mathematics that uses functions and relations to study geometric phenomena (e.g., the description of eclipses and other conic sections in the coordinate plane by quadratic equations).

Angle: The space or opening between a pair of lines, called rays. A figure formed when two rays meet at a single point, called the 'vertex'.

Area: Measure of the amount of surface within the perimeter of a flat (two-dimensional) figure. Area is always measured in 'square' units.

Arithmetic Sequence: A sequence of numbers in which each term except the first term is the result of adding the same number, called the common difference, to the preceding term.

ASA congruence: Angle-side-angle congruence; when two triangles have corresponding angles and sides that are congruent, the triangles themselves are congruent.

Associative [grouping] property: A mathematical rule stating that when more than two numbers are added or multiplied, the result will be the same no matter how the numbers are grouped:
$(a + b) = c = a + (b + c)$ or $(a \times b) \times c = a \times (b \times c)$. (See Table 2)

Assumption: A fact of statement (as a proposition, axiom, postulate, or notion) taken for granted**.**

Asymptotes: A line that continually approaches a given curve but does not meet it at any finite distance.

Attribute: A common feature of a set of figures.

Average: Add the items to average, and then divide this total by the number of items in the list.

B

Bar graph: Visual presentation of data from different sources (e.g., the height or length of bars against the same scale).

Benchmark fraction: A common fraction against which other fractions can be measured, such as $\left(\frac{1}{2}\right)$.

Binomial Theorem: A method for distributing powers of binomials**.**

Bisect: To divide into two equal parts.

Bivariate data: Pairs of linked numerical observations (e.g., a list of heights and weights for each player on a football team).

Box plot: A graphic method that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

C

c = nr = c (total cost) = n (number of items) r (rate or price per item) to find the total cost of a number of similar items. A formula is a standardized (consistent) equation to use in order to answer the same type of question.

Calculus: The mathematics of change and motion. The main concepts of calculus are limits, instantaneous rates of change, and areas enclosed by curves.

Capacity: The maximum amount of number that can be contained or accommodated, e.g., a just with a one-gallon *capacity*; the auditorium was filled to *capacity*.

Cardinal number: A number (as 1, 5, 15) that is used in simple counting and that indicates how many elements there are in a set.

Cartesian plane: A coordinate plane with perpendicular coordinate axes.

Cavalieri's principle: A method, with formula given below, of finding the volume of any solid for which cross-sections by parallel planes have equal areas. This includes, but is not limited to, cylinders and prisms. Formula: Volume = Bh, where B is the area of a cross-section and h is the height of the solid.

Chart: The visual organization and presentation of data in rows and columns.

Circle: The curve formed by all the points in a plane that are the same distance (radius) from a given point (center) and there is 360° in a circle.

Circle graph: A visual presentation of data showing parts of a whole (the circle) using percents, decimals, or fractions.

Circumference: The distance around the edge of a circle.

Coefficient: Any of the factors of a product considered in relation to a specific factor.

Commutative [order] property: A mathematical rule stating that the order in which numbers are added (or multiplied) does not change the sum (or product): $a + b = b + a$ or $a \times b = b \times a$. (See Table 3)

Combination: An arrangement of objects in which order does not matter.

Comparing: Determining which number is greater; arranging numbers in order; using equality and inequality symbols ($=$, $<$, $>$, $\leq$, $\geq$).

Complementary angles: Two angles for which the sum of their measures is 90°.

Complex fraction: A fraction A/B where A and/or B are fractions (B nonzero).

Complex number: A number that can be written as the sum or difference of a real number and an imaginary number. (See Illustration 1)

Complex plane: The coordinate plane used to graph complex numbers.

Compose numbers: a) Given pairs, triples, etc. of numbers, identify sums or products that can be computed; b) Each place in the base ten place value is composed of ten units of the place to the left (e.g., one hundred is composed of ten bundles of ten, one ten is composed of ten ones, etc.)

Compose shapes: Join geometric shapes without overlaps to form new shapes**.**

Composite number: A whole number that has more than two factors.

Computation algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See *algorithm* and *computation strategy.*

Computation strategy: Purposeful manipulations that may be chosen for specific problems that gives the correct result in every case when the steps are carried out correctly. See *computation algorithm.*

Congruent: Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations); having the same size and shape; equal.

Congruent angles: Angles that have equal measures.

Congruent figures: Figures that have the same shape and size.

Conic Section:

> **Ellipse:** A curved line forming a closed loop, where the sum of the distances from two points (foci) to every point on the line is constant.
>
> **Hyperbola:** A symmetrical open curve formed by the intersection of a cone with a plane at a smaller angle with its axis than the side of the cone.
>
> **Parabola:** The set of points that is the same distance away from a single point called the focus and a line called the directrix. Parabolas are symmetric, and their lines of symmetry pass through the vertex. The vertex is the point which has the smallest distance between both the focus and the directrix.

Conjugate: The result of writing sum of two terms as a difference, or vice versa. For example, the conjugate of x - 2 is x + 2.

Convert: To change.

Conversions: The equivalency of change from one unit of measurement to another.

Coordinate graph: A system for finding the location of a point on a flat surface called a plane, and two numbers, an x-coordinate and a y-coordinate, name each point on the grid.

Coordinate plane: A plane in which two coordinate axes are specified (i.e., two intersecting directed straight lines, usually perpendicular to each other, and usually called the x-axis and y-axis). Every point in a coordinate plane can be described uniquely by an ordered pair of numbers, the coordinates of the point with respect to the coordinate axes.

Coordinate system: A set of points formed by a grid with a horizontal x-axis and vertical y-axis. The line that extends horizontally in both directions through the origin is the x-axis. The y-axis extends vertically in both directions through this central point.

Cosine: A trigonometric function that for an acute angle is the ratio between a leg adjacent to the angle when the angle is considered part of a right triangle and the hypotenuse.

Counting number: A number used in counting objects (e.g., a number from the set 1, 2, 3, 4, 5….) (See Illustration 1)

Counting on: A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Cube: A rectangular solid with six square sides (faces-all edges of equal length); raise to the third power (3) - multiply a number times itself 3 times (n x n x n = n3).

Cylinder: A solid (three-dimensional) figure with two congruent circular bases and straight sides.

D

d is the **distance,** r is the rate, and t is the time (d = rt).

Decimal: A fraction expressed in the place value system to the right of the decimal point.

Decimal expansion: Writing a rational number as a decimal.

Decimal fraction: A fraction (as 0.25 = $\frac{25}{100}$ or 0.025 = $\frac{25}{1000}$) or mixed number (as 3.025 = 3 $\frac{25}{1000}$) in which the denominator is a power of ten, usually expressed by the use of the decimal point.

Decimal number: Any real number expressed in base 10 notation, such as 2.673

Decompose numbers: Given a number, identify pairs, triples, etc. of numbers that combine to form the given number using subtraction and division.

Decompose shapes: Given the geometric shape, identify geometric shapes that meet without overlap to form the given shape**.**

Denominator: The bottom number of a fraction.

Diagonal: A line segment drawn between the vertices of two nonadjacent sides of a figure that has four or more straight sides.

Diameter: The distance across a circle that passes through the middle.

Dilation: A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Digit: a) Any of the Arabic numerals 1 to 9 and usually the symbol 0; b) One of the elements that combine to form numbers in a system other than the decimal system.

Directrix: A fixed curve with which a generatrix maintains a given relationship in generating a geometric figure; specifically: a straight line the distance to which from any point in a conic section is in fixed ratio to the distance from the same point to a focus.

Distance formula: Distance (d) = rate (r) x time (t) or $d = rt$

Distributive property: Means to multiply a factor by a sum of terms, multiply the factor by each term in parentheses, than combine the products.

Domain: The set of all possible input values (usually x) which allows the function formula to work.

Dot plot: See *line plot*.

E

Equation: The mathematical statement that says two expressions are equal.

Equidistant: A point the same distance from two or more other points.

Equilateral: Having equal sides.

Equilateral triangle: Triangle with three congruent sides; an equilateral triangle also has three congruent angles, each measuring 60°.

Evaluate: An expression substitute known or given values for the variables in an algebraic expression (perform the operations in the order of operations to obtain the solution).

Expanded form: A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example: 643 = 600 + 40 + 3

Expected value: For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Exponent: The number that indicates how many times the base is used as a factor (e.g., in $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times).

Exponential function: A function of the form $y = a \cdot b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$. The variables do not have to be x and y. For example, $A = 3.2 \cdot (1.02)^t$ is an exponential function.

Expression: A mathematical phrase that combines operations, numbers, and/or variables (e.g., $3^2 - a$).

F

Factor: To find the number of elements that exist within that particular number.

Factoring: Finding the algebraic terms or expressions (called factors) that when multiplied will result in a certain product.

First quartile: For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] See *median*, *third quartile*, and *interquartile range*.

FOIL: A method for multiplying algebraic factors with more than one term. FOIL stands for first-outer-inner-last.

Formula: An equation that helps solve math problems.

Fraction: A way to show part (the numerator or top number) of a whole (the denominator or bottom number). Digits grouped above and below a division bar is a ratio.

Function: An algebraic rule involving two variables in which for every value of the first variable (x), there is a unique value of the second variable (y).

Function notation: A notation that describes a function. For a function f, when x is a member of the domain, the symbol f(x) denotes the corresponding member of the range (e.g., $f(x) = x + 3$).

Fundamental Theorem of Algebra: The theorem that establishes that, using complex numbers, all polynomials can be factored. A generalization of the theorem asserts that any polynomial of degree *n* has exactly *n* zeros, counting multiplicity.

G

Geometric sequence (progression): An ordered list of numbers that has a common ratio between consecutive terms (e.g., 2, 6, 18, 54….).

Graph: A visual representation comparing data from different sources or over time.

H

Histogram: A type of bar graph used to display the distribution of measurement data across a continuous range.

Hypotenuse: The side of a right triangle opposite the right angle. The hypotenuse is always the longest side.

Horizontal axis: A scale that runs along the bottom or left to right on a graph or coordinate grid; the x-axis.

Hypotenuse of a right triangle: The line that is opposite the right angle. It is represented by the letter **c** in the formula $c^2 = a^2 + b^2$.

I

Identity property of 0: The sum of zero and any number is that given number. (See Table 3)

Imaginary number: Complex numbers with no real terms, such as 5*i*. (See Illustration 1)

Improper fraction: A fraction in which the value of the numerator is greater than or equal to the value of the denominator.

Independently combined probability models: Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Inequalities: Statements that two amounts are not equal.

Integers: All whole numbers, positive, negative, and zero.

Intercept: The point where the line crosses the y-axis. This is found by setting x = O.

Interest: The amount of money that is paid or charged for the use of money over a certain period of time. The simple interest formula is (p)rincipal x (r)ate x (t)ime or interest = prt.

Interquartile range: A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See *first quartile* and *third quartile*.

Inverse: Opposite; addition and subtraction are inverse operations, as are multiplication and division.

Inverse function: A function obtained by expressing the dependent variable of one function as the independent variable of another; that is the inverse of $y - f(x)$ is $x = f^{-1}(y)$.

Irrational number: A number that cannot be expressed as a quotient of two integers (e.g., $\sqrt{2}$). It can be shown that a number is irrational if and only if it cannot be written as a repeating or terminating decimal.

Isosceles: Having two sides equal.

Isosceles triangle: A triangle in which two sides have the same length, and the two angles opposite the equal sides have the same measure.

L

Law of Cosines: An equation relating the cosine of an interior angle and the lengths of the sides of a triangle.

Law of Sines: Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides.

Leg in a right triangle: One of the two sides that form the right angle.

Like terms: The same variable or variables raised to the same power.

Line graph: Visual presentation of data as a line on a grid, often showing change over time (a trend).

Linear association: Two variables have a linear association if a scatter plot of the data can be well-approximated by a line.

Linear equation: An equation that does not contain a variable to any power (exponent) greater than 1; an equation whose graph is a straight line.

Line plot: A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line; also known as a dot plot.

Linear function: A mathematical function in which the variables appear only in the first degree, are multiplied by constants, and are combined only by addition and subtraction, multiplied by constants, and are combined only by addition and subtraction. For example: $f(s) = Ax + By + C$

Logarithm: The exponent that indicates the power to which a base number is raised to produce a given number. For example, the logarithm of 100 to the base 10 is 2.

Logarithmic function: Any function in which an independent variable appears in the form of a logarithm; they are the inverse functions of exponential functions.

M

Magnitude: The length of the vector.

Matrix: A rectangular array of numbers or variables. Plural: matrices.

Maxima: y_0 is the "absolute maximum" of f(x) on I if and only if y_0 >= f(x) for all x on I.

Mean: A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list, the average. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation: A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Measure of variability: A determination of how much the performance of a group deviates from the mean or median; most frequently used measure is standard deviation.

Median: A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list; or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11**.**

Metric system of measurement: A measurement system used throughout most of the world that is based on the powers of ten. Common units are meters (unit of length), grams (unit of weight or mass), and liters (unit of volume).

Midline: In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Minima: y_0 is the "absolute minimum" of f(x) on I if and only if y_0 <= f(x) for all x on I.

Mixed number: A quantity expressed as a whole number and a proper fraction.

Mode: A list of data is the number occurring most often.

Model: A mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real-world or mathematical objects, properties, actions, or relationships.

Modulus of a complex number: The distance between a complex number and the origin on the complex plane. The absolute value of $a + bi$ is written $|a + bi|$, and the formula for $|a + bi|$ is $\sqrt{a^2 + b^2}$. For a complex number in polar form, $r(\cos\theta + i \sin \theta)$, the modulus is r.

Multiplication and division within 100: The multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses: Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

N

Negative number: A number to the left of zero on a number line; a number less than zero in value; used to show a decrease, a loss, or downward direction; may be preceded by a minus sign.

Network: a) A figure consisting of vertices and edges that shows how objects are connected, b) A collection of points (vertices), with certain connections (edges) between them.

Non-adjacent angles: Angles that do not share a common ray; they may or may not share a common vertex.

Number line: A line divided into equal segments (intervals) by points corresponding to integers, fractions, or decimals; points to the right of 0 are positive; those to the left are negative. Signed numbers extend indefinitely in both directions on the number line.

Number line diagram: A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Numeral: A symbol or mark used to represent a number.

Numerator: The top number in a fraction.

O

Obtuse angle: Measures more than 90° but less than 180°.

Order of Operation: The part of a mathematical expression that must be computed first, second, third, and so on. It is the sequence, agreed upon by mathematicians, for performing mathematical operations: 1. Operations in grouping symbols, 2. Exponents and roots, 3. Multiplication and division from left to right, 4. Addition and subtraction from left to right.

Order pair: A pair is of numbers that names a point on a coordinate graph; presented in parentheses (the x- coordinate, the y-coordinate).

Ordinal number: A number designating the place (as first, second, or third, etc.) occupied by an item in an ordered sequence.

Operation: An action preformed to one or more numbers to produce an answer: addition, subtraction, multiplication, division, exponents, and roots.

Origin: The point at which the x-axis and y-axis in a coordinate graph intersect; the point represented by the ordered pair (0,0).

Outlier: An element of a data set that distinctly stands out from the rest of the data; it is significantly larger or smaller than the other numbers in the data set.

P

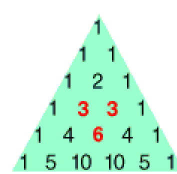
Parallel lines: Two lines on the same plane that do not intersect.

Parallelogram: A quadrilateral with both pairs of opposite sides parallel; opposite sides are of equal length, and opposite angles are of equal measure.

Parentheses: What surrounds the mathematical expression (24 - 16) + 67 = 75.

Partition: A process of dividing an object into parts.

Pascal's triangle: A triangular arrangement of numbers in which each row starts and ends with 1, and each other number is the sum of the two numbers above it.



Percent: The number of parts per hundred. A percent is another way of expressing a fraction with a denominator of 100.

Percent rate of change: A rate of change expressed as a percent. Example: If a population grows from 50 to 55 in a year, it grows by $\frac{5}{50}$ = 10% per year.

Period (of trig functions): The distance between any two repeating points on the function**.**

Periodic phenomena: Naturally recurring events. For example, ocean tides and machine cycles.

Perimeter: The distance around a flat (2-D) figure; the sum of the lengths of all the sides of a flat figure.

Permutation: Different arrangements of a group of items where order matters.

Perpendicular lines: Two lines that intersect forming adjacent right angles.

Pi (π): The constant ratio of the circumference of a circle to the diameter; approximately 3.14 or $\frac{22}{7}$.

Picture graph: A graph that uses pictures to show and compare information.

Plane: A set of points that forms a flat surface.

Polynomial: The sum or difference of terms which have variables raised to positive integer powers and which have coefficients that may be real or complex. The following are all polynomials: $5x^3 - 2x^2 + x - 13$, $x^2y^3 + xy$, and $(1 + I)a^2 + ib^2$

Polynomial function: Any function whose value is the solution of a polynomial.

Positive number: A number to the right of zero on a number line; a number greater than zero in value, used to show an increase, a gain, or upward direction; may be preceded by a plus sign.

Postulate: A statement accepted as true without proof.

Prime factorization: A number written as the product of all its prime factors.

Prime number: A whole number greater than 1 whose only factors are 1 and itself.

Probability: A number (whole, fraction, decimal, or ratio) that shows how likely it is that an event will happen; chance.

Probability distribution: The set of possible values of a random variable with a probability assigned to each.

Probability model: A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See *uniform probability model*.

Product: The answer to a multiplication problem.

Proof: A method of constructing a valid argument, using deductive reasoning.

Properties of equality: See Table 4.

Properties of inequality: See Table 5.

Properties of operations: A set of rules for determining the order of operations. (See Table 3)

Proportion: An equation that states that two ratios (fractions) are equal (e.g., $\frac{4}{8} = \frac{1}{2}$ or 4:8 = 1:2).

Pythagorean Theorem: The square of the hypotenuse is equal to the sum of the squares of the other two sides (legs): $a^2 + b^2 = c^2$

Q

Quadrant: One-fourth of a coordinate grid, formed by the intersecting axes.

Quadratic equation: An equation that contains a variable raised to the second power; there may be two solutions to a quadratic equation. Some examples are $y = 3x^2 - 5x^2 + 1$, $x^2 + 5xy + y^2 = 1$, and $1.6a^2 + 5.9a - 3.14 = 0$

Quadratic expression: Contains a variable raised to the second power.

Quadratic function: A function that can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of this function is a parabola.

Quadratic polynomial: A polynomial where the highest degree of any of its terms is 2.

Quadrilateral: A polygon with four sides and four angles.

Quotient: Answer to a division problem; the amount in each part of the whole.

R

Radian: A unit of angular measurement such that there are 2 pi radians in a complete circle. One radian = 180/pi degrees. One radian is approximately 57.3^{o}.

Radical: The √ symbol, which is used to indicate square roots or nth roots.

Radius: The distance from the middle of the circle to the edge. Radius = $\frac{1}{2}$ diameter

Random: Selected by chance, with no outcome more likely than any other.

Random sampling: A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects.

Random variable: An assignment of a numerical value to each outcome in a sample space.

Range: The spread between the lowest number and the highest number.

Rate: A ratio of two different kinds of units used to show a relationship:
miles gallon, miles per hour; the percent relationship of the part to the base in a percent problem.

Ratio: A comparison of two numbers used to show a relationship or pattern.

Rational expression: A quotient of two polynomials with a non-zero denominator.

Rational number: A number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$. The rational numbers include the integers. (See Illustration 1)

Ray: Part of a line having only one endpoint (one side of an angle).

Real number: A number from the set of numbers consisting of all rational and all irrational numbers. (See Illustration 1)

Rectangle: A four-sided figure in which the opposite sides are equal in length.

Rectangular solid: A three-dimensional figure with six sides, all of which are rectangles.

Rectilinear figure: A polygon all angles of which are right angles.

Recursive pattern or sequence: A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.

Reflection: A type of transformation that flips points about a line, called the *line of reflection.* Taken together, the image and the pre-image have the line of reflection as a line of symmetry.

Reflex angle: An angle that measures more than 180° but less than 360°

Relative frequency: The empirical counterpart of probability. If an event occurs N' times in N trials, its relative frequency is N'/N.

Remainder Theorem: If $f(x)$ is a polynomial in x then the remainder on dividing $f(x)$ by $x - a$ is $f(a)$.

Repeating decimal: A decimal number that continues infinitely, repeating a pattern of digits.

Residual: The observed value minus the predicted value. It is the difference of the results obtained by observation, and by computation from a formula.

Rhombus: A parallelogram with four equal sides.

Right angle: An angle that makes a squared corner that measures exactly 90°.

Rigid motion: A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Rise: The change in the y value in vertical distance.

Rotation: A type of transformation that turns a figure about a fixed point, also called the center of rotation.

Rounding: Using an approximation for a number.

Run: The horizontal change in the x value.

S

Sample space: In a probability model for a random process, a list of the individual outcomes that are to be considered.

SAS congruence: Side-Angle-Side congruence. When two triangles have corresponding sides and the angles formed by those sides are congruent, the triangles and congruent.

Scalar: A scalar is a quantity, which has only magnitude but no direction.

Scale factor: The ratio of corresponding lengths of the sides of two similar figures.

Scalene: No equal sides and no equal angles.

Scalene triangle: A triangle in which all three sides have different lengths and all three angles have different measurements.

Scatter plot: A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

Sequence / progression: A set of elements ordered so that they can be labeled with consecutive positive integers starting with 1 (e.g., 1, 3, 9, 27, 81). In this sequence, 1 is the first term, 3 is the second term, 9 is the third term, and so on.

Scientific notation: A way of writing very large numbers and very small decimals in which the numbers are expressed as the product of a number between 1 and 10 and a power of 10 (e.g., $562 = 5.62 \times 10^2$).

Signed numbers: Positive and negative numbers; often used to show quantity, distance, or direction.

Similar figures: Figures in which the corresponding angles and sides are proportionate; figures having the same shape, but different sizes.

Similarity transformation: A rigid motion followed by a dilation.

Simple interest: Fee charged for borrowing money (or earned for investing money) for a particular period of time; simple interest = principal x rate x time.

- **Principal** is the amount of the purchase.
- **Rate** is the percent you will pay for the loan.
- **Time** is how long you are going to borrow the money.

Simplified: Means to reduce to lowest terms; or perform all operations within a mathematical expression.

Sine: The trigonometric function that for an acute angle is the ratio between the leg opposite the angle when the angle is considered part of a right triangle and the hypotenuse.

Slope: Refers to the steepness of the line or rise over run.

Slope-intercept formula: An equation of a line that takes the following form: $y = mx + b$, where m is the slope and b is the y-intercept.

Solution set: Not one number but a set of whole numbers.

Solving percents: The base represents the whole amount in a percent problem. The rate is the percent relationship of the part to the base in a percent problem. The part is a portion of the whole or base in a percent problem.

Square: A figure with 4 right angles (a type of rectangle) and 4 sides of equal length (a special rhombus); numerical operation in which a number is multiplied by itself, represented by the exponent 2.

Square root: A number is the number that when multiplied by itself equals the given number.

SSS Congruence: Side-Side-Side Congruence. When two triangles have corresponding sides that are congruent, the triangles are congruent.

Straight angle: Measures exactly 180°, every straight angle.

Supplementary angles: Two angles for which the sum of their measures is 180°.

Symmetry: The quality of being made up of exactly similar parts facing each other or around an axis.

T

Tangent: a) Meeting a curve or surface in a single point of a sufficiently small interval is considered; b) The trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle and the leg adjacent to the angle when the angle is considered part of a right triangle.

Tape diagram: A drawing that looks like a segment of tape, used to illustrate number relationships; also known as a strip diagram, bar model, fraction strip, or length model.

Terms: A single number, a variable, or numbers and variable multiplied together.

Terminating decimal: A decimal is terminating if its repeating digit is 0. A terminating decimal is the decimal form of a rational number.

Third quartile: For a data set with median M, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: *median, first quartile, interquartile range*.

Transformation: A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the pre-image) and the points in another geometric object (the image). Reflections, rotations, translations, and dilations are particular examples of transformations.

Transitivity principle for indirect measurement: If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Translation: A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size.

Transversal: A line that crosses two or more parallel lines.

Trapezoid: A quadrilateral with two sides that are parallel.

Triangles: (Δ) is the symbol for a triangle. A triangle has three sides and has three angles, which together add up to 180 and are identified by vertices (points), e.g., (ΔABC).

Trigonometric function: A function (as the sine, cosine, tangent, cotangent, secant, or cosecant) of an arc or angle most simply expressed in terms of the ratios of pairs of sides of a right angled triangle.

Trigonometry: The study of triangles, with emphasis on calculations involving the lengths of sides and the measures of angles.

U

Uniform probability model: A probability model which assigns equal probability to all outcomes. See also: *probability model*.

Unit fraction: A fraction with a numerator of 1, such as $\frac{1}{3}$ or $\frac{1}{5}$.

Unit price: The cost of one item.

Unit rate: The rate for one unit of a given quantity; has a denominator of zero.

V

Valid: a) Well-grounded or justifiable; being at once relevant and meaningful (e.g., a *valid* theory); b) logically correct.

Variable: a) Any letter used to stand for a number; b) A quantity that can change or that may take on different values. It refers to the letter or symbol representing such a quantity in an expression, equation, inequality, or matrix.

Variable expression: A combination of numbers, variables, and mathematical operations arranged in a meaningful order.

Vector: A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Vertex: The point at which two or more line segments or sides of a figure meet; the point at which the two rays that form an angle meet.

Vertical angles: The angles across from each other when two lines intersect or cross; also called opposite angles.

Vertical axis: The scale that runs along the side or top to bottom on a graph or coordinate grid (the y- axis).

Visual fraction model: A tape diagram, number line diagram, or area model.

Volume: A measure of the amount of space inside a three-dimensional or solid figure. Volume is always measured in cubic (^{3}) units.

W

Whole numbers: The numbers 0, 1, 2, 3,… (See Illustration 1)

X

(x,y) coordinates: An ordered pair; enclosed in parentheses, separated by a comma.

x-axis: The horizontal axis in a coordinate graph.

x-coordinate: The first number in an ordered pair; the distance from the origin along the x-axis.

x-intercept: The point at which a line crosses the x-axis on a coordinate graph; the ordered pair (x,0).

Y

y-axis: The vertical axis in a coordinate graph.

y-coordinate: The second number in an ordered pair, the distance from the origin along the y-axis.

y-intercept: The point at which a line crosses the y-axis on a coordinate graph; the ordered pair (0,y).

Tables and Illustrations of Key Mathematical Properties, Rules, and Number Sets

TABLE 1. Common addition and subtraction situations.[21]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[22]
Put Together/ Take Apart[23]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

[21] Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32–33).
[22] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.
[23] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[24]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[24] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations.[25]

	Unknown Product **3 × 6 = ?**	**Group Size Unknown** (“How many in each group?” Division) **3 × ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown** (“How many groups?” Division) **? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[26] Area[27]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

[25] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[26] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[27] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

TABLE 3. The properties of operations.

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

TABLE 4. The properties of equality.

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

TABLE 5. The properties of inequality.

Here *a*, *b*, and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

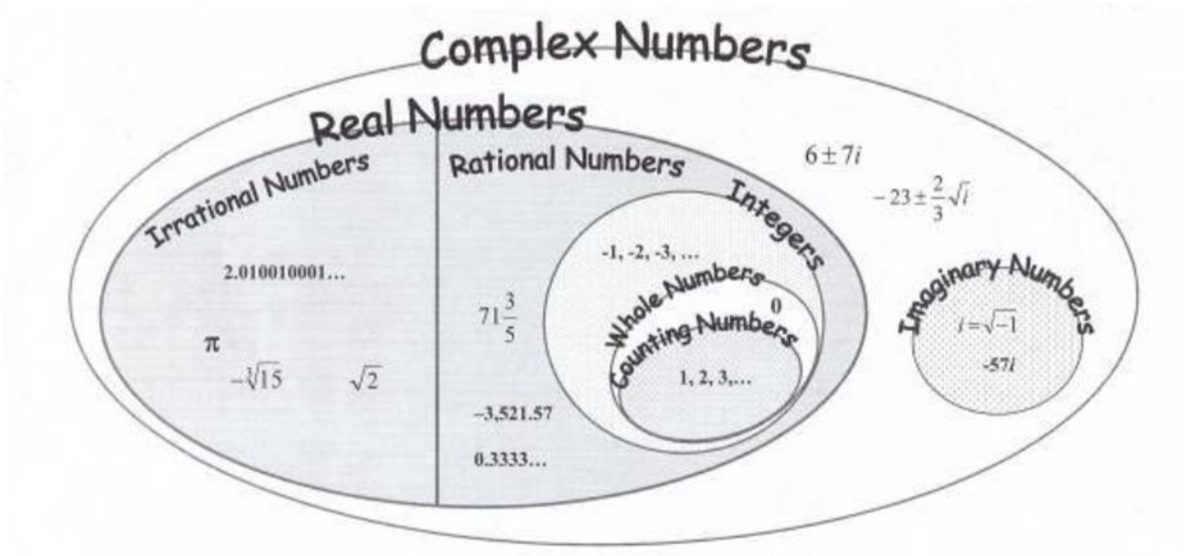
If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.

ILLUSTRATION 1. The Number System.

The Number System is comprised of number sets beginning with the Counting Numbers and culminating in the more complete Complex Numbers. The name of each set is written on the boundary of the set, indicating that each increasing oval encompasses the sets contained within. Note that the Real Number Set is comprised of two parts: Rational Numbers and Irrational Numbers.



Appendix A – Financial Literacy Resources

National Standards for Adult Financial Literacy Education
https://financiallit.org/wordpress/resources/national-standards/
The National Standards for Adult Financial Literacy Education identify the personal finance knowledge and skills an adult should possess. The Institute for Financial Literacy® contends that all adults who receive financial literacy education should have, at a minimum, the knowledge and ability to competently perform the basic tasks of managing their personal finances.

Practical Money Skills for Life https://www.practicalmoneyskills.com/
To help consumers and students of all ages learn the essentials of personal finance, Visa has partnered with leading consumer advocates, educators, and financial institutions to develop the Practical Money Skills program. At this website you can access free educational resources, including personal finance articles, games and lesson plans.

Money Smart for Young Adults
https://www.fdic.gov/consumers/consumer/moneysmart/young.html
The FDIC's Money Smart for Young Adults curriculum helps youth ages 12-20 learn the basics of handling their money and finances, including how to create positive relationships with financial institutions. Equipping young people in their formative years with the basics of financial education can give them the knowledge, skills, and confidence they need to manage their finances once they enter the real world. Money Smart for Young Adults consists of eight instructor-led modules. Each module includes a fully scripted instructor guide, participant guide, and overhead slides. The materials also include an optional computer-based scenario that allows students to complete realistic exercises based on each module.

Money Skill https://afsaef.org/
MoneySKILL is a free online reality based personal finance course for young adults developed by the AFSA Education Foundation. This interactive curriculum is aimed at the millions of high school and college students who graduate each year without a basic understanding of money management fundamentals. The course is designed to be used as all or part of a grade for courses in economics, math, social studies or where personal finance are taught. Students experience the interactive curriculum as both written text and audio narration. In addition, frequent quizzes test their grasp of each and every concept. The 36-module curriculum, with pre- and post -tests, covers the content areas of *income, expenses, assets, liabilities and risk management*. A life simulation module asks students to project their own financial life expectancies in areas such as employment, housing, transportation, education, marriage, family and retirement. The life simulation allows students to incorporate the *MoneySKILL* personal finance concepts into their everyday lives, thus providing them with knowledge and skills that will last a lifetime.

More financial literacy resources available at: https://www.sabes.org/resources/financial-literacy.htm

Appendix B – Standards for Mathematical Practice Reference Sheet[28]

<table>
<tr><th>Summary of Standards for Mathematical Practice</th><th>Questions to Develop Mathematical Thinking</th></tr>
<tr><td>1. Make sense of problems and persevere in solving them.

Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.
Plan a solution pathway instead of jumping to a solution.
Can monitor their progress and change the approach if necessary.
See relationships between various representations.
Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
Continually ask themselves, “Does this make sense?” Can understand various approaches to solutions.
</td><td>
How would you describe the problem in your own words?
How would you describe what you are trying to find?
What do you notice about...?
What information is given in the problem?
Describe the relationship between the quantities.
Describe what you have already tried. What might you change?
Talk me through the steps you’ve used to this point.
What steps in the process are you most confident about?
What are some other strategies you might try?
What are some other problems that are similar to this one?
How might you use one of your previous problems to help you begin?
How else might you organize...represent... show...?
</td></tr>
<tr><td>2. Reason abstractly and quantitatively.

Make sense of quantities and their relationships.
Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.
Understand the meaning of quantities and are flexible in the use of operations and their properties.
Create a logical representation of the problem.
Attends to the meaning of quantities, not just how to compute them.
</td><td>
What do the numbers used in the problem represent?
What is the relationship of the quantities?
How is _______ related to _______?
What is the relationship between _______ and _______?
What does _______ mean to you? (e.g., symbol, quantity, diagram)
What properties might we use to find a solution?
How did you decide in this task that you needed to use...?
Could we have used another operation or property to solve this task? Why or why not?
</td></tr>
</table>

[28] Wichita Public School – Mathematics Department, May 2016, https://achievethecore.org/aligned/putting-the-math-practices-into-practice/

<table>
<tr><th>Summary of Standards for Mathematical Practice</th><th>Questions to Develop Mathematical Thinking</th></tr>
<tr><td>3. Construct viable arguments and critique the reasoning of others.
• Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
• Justify conclusions with mathematical ideas.
• Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
• Ask clarifying questions or suggest ideas to improve/revise the argument.
• Compare two arguments and determine correct or flawed logic.</td><td>▪ What mathematical evidence would support your solution?
▪ How can we be sure that...? / How could you prove that...?
▪ Will it still work if...?
▪ What were you considering when...?
▪ How did you decide to try that strategy?
▪ How did you test whether your approach worked?
▪ How did you decide what the problem was asking you to find? (What was unknown?)
▪ Did you try a method that did not work? Why didn’t it work? Would it ever work? Why or why not?
▪ What is the same and what is different about...?
▪ How could you demonstrate a counter-example?</td></tr>
<tr><td>4. Model with mathematics.
• Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize).
• Apply the math they know to solve problems in everyday life.
• Are able to simplify a complex problem and identify important quantities to look at relationships.
• Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.
• Reflect on whether the results make sense, possibly improving/revising the model.
• Ask themselves, “How can I represent this mathematically?”</td><td>▪ What number model could you construct to represent the problem?
▪ What are some ways to represent the quantities?
▪ What’s an equation or expression that matches the diagram..., number line.., chart..., table..?
▪ Where did you see one of the quantities in the task in your equation or expression?
▪ What math do you know that you could use to represent this situation?
▪ What assumptions do you have to make to solve the problem?
▪ What formula might apply in this situation?</td></tr>
</table>

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	▪ What mathematical tools could we use to visualize and represent the situation? ▪ What information do you have? ▪ What do you know that is not stated in the problem? ▪ What approach are you considering trying first? ▪ What estimate did you make for the solution? ▪ In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? ▪ What can using a _______ show us that _______ may not? ▪ In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	▪ What mathematical terms apply in this situation? ▪ How did you know your solution was reasonable? ▪ Explain how you might show that your solution answers the problem. ▪ Is there a more efficient strategy? ▪ How are you showing the meaning of the quantities? ▪ What symbols or mathematical notations are important in this problem? ▪ What mathematical language..., definitions..., properties can you use to explain...? ▪ How could you test your solution to see if it answers the problem?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	▪ What observations do you make about...? ▪ What do you notice when...? ▪ What parts of the problem might you eliminate..., simplify...? ▪ What patterns do you find in...? ▪ How do you know if something is a pattern? ▪ What ideas that we have learned before were useful in solving this problem? ▪ What are some other problems that are similar to this one? ▪ How does this relate to...? ▪ In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	▪ Will the same strategy work in other situations? ▪ Is this always true, sometimes true or never true? ▪ How would we prove that...? ▪ What do you notice about...? ▪ What is happening in this situation? ▪ What would happen if...? ▪ Is there a mathematical rule for...? ▪ What predictions or generalizations can this pattern support? ▪ What mathematical consistencies do you notice?